Supersolid Vortex Crystals in Rydberg-Dressed Bose-Einstein Condensates

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We study rotating quasi-two-dimensional Bose–Einstein condensates, in which atoms are dressed to a highly excited Rydberg state. This leads to weak effective interactions that induce a transition to a mesoscopic supersolid state. Considering slow rotation, we determine its superfluidity using quantum Monte Carlo simulations as well as mean field calculations. For rapid rotation, the latter reveal an interesting competition between the supersolid crystal structure and the rotation-induced vortex lattice that gives rise to new phases, including arrays of mesoscopic vortex crystals.

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Superfluidity, i.e., the frictionless flow of a liquid, is one of the most spectacular manifestations of quantum mechanical behavior on a macroscopic scale. Generally, superfluidity can be characterized from the response of a many-body system to a slow, externally imposed rotation. A classical fluid enclosed in a rotating vessel will be dragged along and eventually rotate with it. A superfluid, on the other hand will, due to a lack of viscosity, remain stationary and for sufficiently fast rotation form quantized vortices that arrange on a regular lattice [1,2]. Superfluidity has long been speculated to occur even in solid states of matter, combining the seemingly antithetical qualities of crystalline order and nondissipative flow [3–5]. Recent experimental evidence for such a peculiar supersolid phase in 4He crystals [6,7] has sparked an intense debate about its physical origin, which however remains controversial [8].

On the other hand, ultracold gases have emerged as a powerful laboratory to study a diverse range of many-body problems, including discrete supersolid states in optical lattices [9]. A promising route to the observation of continuous supersolids has been recently laid out [10,11], on the basis of off-resonant dressing of atomic Bose–Einstein condensates (BECs) to high-lying Rydberg states [12–16]. The effective atomic interactions resulting from such a Rydberg dressing provide a clean realization of a simple model for supersolidity [17]. Given the range of available techniques for accurate probing of BECs [18] and recent advances in the manipulation of cold Rydberg atoms [19], this approach holds promise for the observation of supersolidity under well-defined and highly controllable conditions.

In this Letter, we explore the possibility for creating and detecting mesoscopic supersolids in confined, Rydberg-dressed BECs by probing their response to forced trap rotations. For infinitesimally small rotation frequencies, large-scale simulations demonstrate that crystalline order and superfluidity (see Fig. 1) persist at well-accessible temperatures, enabling supersolid creation in cold atom experiments. Combining the results from first-principle quantum Monte Carlo (QMC) simulations and mean field calculations, we reveal a universal behavior of the crystalline ordering and the superfluidity that enables simple estimates of the system behavior over a wide range of

![Figure 1](image-url)

FIG. 1 (color online). (a) Schematics of the considered setup in which the ground states (|g⟩) of condensed atoms are off-resonantly coupled to high-lying Rydberg states (|e⟩) with a large laser detuning Δ and a small Rabi frequency Ω ≪ Δ. The resulting effective interaction, shown in (b) can lead to the formation of mesoscopic supersolids in a quasi-two-dimensional condensate, exemplarily shown in panels (c) and (d). The extended plateau of the histogram P(k) of k-particle permutations, shown in (c) indicates a large superfluid fraction, whereas the density shown in (d) demonstrates crystallization. The results correspond to N = 3000 rubidium atoms confined in a trap with ωz/2π = 125 Hz and dressed to Rb(50S1/2) Rydberg states with realistic laser parameters of Δ/2π = 75 MHz and Ω/2π = 2.2 MHz. The depicted QMC results yield a large superfluid fraction f_s = 0.51(4) for an experimentally accessible temperature of T = 42 nK.
of many atoms in each blockade sphere, forming a solid of liquid droplets with collectively enhanced interactions. On the other hand, the average number of Rydberg excitations in each droplet is only \( \sim 0.04 \), which diminishes Rydberg state decay \([10,13]\) and renders many-body effects that tend to counteract the binary interaction (1) \([26]\), unimportant.

In addition, the QMC simulation yields a large superfluid fraction of \( f_s = 0.5 \), showing that the observed mesoscopic crystal is indeed a supersolid. \( f_s \) corresponds to the fraction of atoms that decouples from a vanishingly small trap rotation, such that \( f_s = 1 - I_{\text{qm}}/I_c \), where \( I_c \) and \( I_{\text{qm}} \) are the classical and the actual (i.e., quantum mechanical) moments of inertia that can be computed with QMC simulations \([27]\).

The finite value of \( f_s \) is also indicated in Fig. 1(c) by the extended plateau of the probability \( P(k) \) that \( k \) particles exchange within the system. \( P(k) \) is finite for all \( k \leq N \) in a superfluid but drops to zero in a classical crystal \([28]\).

The large superfluid fraction arises from the extended inner plateau of the interaction potential (1), which can accommodate a large number of atoms that maintain a sizable superfluid fraction, even in the defect-free droplet crystals discussed in this work. As a result, the superfluidity does not critically depend on the size and specific structure of the mesoscopic crystals and, in particular, stays finite in the thermodynamic limit of infinitely large systems (see Supplemental Material \([29]\)). This behavior is in contrast to mesoscopic assemblies with pure power-law potentials \([30]\) where the superfluidity depends strongly on the cluster geometry and vanishes in the bulk limit of infinite defect-free crystals \([31]\).

The temperature dependence of \( f_s \) is shown in Fig. 2(a) for different atom numbers \( N \) and interaction strengths \( \alpha \) but with the product \( \alpha N = \text{const} \) held constant. Apparently, superfluidity extends to higher temperatures for larger values of \( N \), which is readily understood from simple arguments. As will be explained below, the crystal structure remains unchanged for \( \alpha N = \text{const} \), such that increasing \( N \) merely increases the number of atoms in each droplet. As a result, the system becomes less susceptible to fluctuations, which facilitates exchange between the mesoscopic crystal sites and, hence, leads to larger \( f_s \). Upon rescaling the temperature with \( N \), all data collapse on a single curve \( \text{[inset of Fig. 2(a)]} \). This universal behavior of the crystalline order and superfluidity with respect to \( \alpha N \) and \( T/N \) can be used to predict the system behavior for a wider range of parameters. For instance, at a typical interaction range \( r_c = 2 \mu \text{m} \) and a large number of \( N = 10^4 \) Rb atoms, the calculations of Fig. 2 imply a crystal structure as in Fig. 2(e) with a superfluid fraction of \( f_s = 0.4 \), both persisting at a large temperature of \( T = 350 \) nK.

The local superfluid density \( \rho_s \) is also closely linked to slow trap rotations and can be obtained with QMC simulations from a local area estimator \([32]\) of the nonclassical rotational inertia. In his seminal paper on supersolidity in helium, Leggett \([5]\) proposed a far simpler approach to estimate the superfluid density by considering the response of a
Likewise, Eq. (3) gives the phase diagrams depicted in Fig. 3. In the region of small $\alpha N$ and $\Omega$ (SF in Fig. 3) we find a simple superfluid, with $\rho_s = \rho$ and an unstructured density profile. Upon increasing the rotation frequency above a critical value, vortices start to form and eventually arrange on a regular lattice, whose spacing decreases for increasing rotation frequency and interaction strengths (VL in Fig. 3). For small values of $\alpha N$, the nonlocal character of the corresponding interaction in Eq. (4) is of minor importance, such that the total interaction can be described via an effective contact term $\gamma' = \gamma + 2\pi^2\alpha/(3\sqrt{2} r_c^2)$ (see Supplemental Material [29]). In this regime, we indeed find ordinary vortex lattices determined by $\gamma' N$ [see Figs. 3(a) i, 3(b) i, and 3(c) i]. With increasing $\alpha N$ the finite range of the dressing-induced interaction becomes important and changes the symmetry of the vortex lattice. For small $r_c$ [Fig. 3(a)] the system is already close to the bulk limit, where one finds a transition to a honeycomb density pattern. For large $r_c$, finite size effects become dominant and lead to the formation of concentric rings superimposed on the underlying vortex lattice [Fig. 3(c) ii].

The finite size system also affects the crossover to supersolid states [38]. In the bulk limit, the critical interaction strength $\alpha_{2D} N$ can be linked to a roton instability of the unmodulated ground state [10] and, therefore, decreases with increasing rotation frequency (at larger $\Omega$ it increases...
again due to the decreasing density caused by the centrifugal potential). For small $r_c$ values [Fig. 3(a)], the critical value at $\Omega = 0$ can thus be estimated from a local density approximation (LDA) of the roton instability, which gives $r_c^{-4} \alpha N [\hat{\psi}(0)]^2 = 31.9 + 6.1 \gamma$ and for the parameters of Fig. 3(a), $r_c^{-4} \alpha N = 9.3 \times 10^4$. For larger $r_c$ values, the supersolid transition is preceded by the aforementioned ring formation. Assuming that the transition is initiated by a one-dimensional roton instability of the innermost ring, one obtains $r_c^{-4} \alpha_{1D} N = 6.7 \times 10^4$ and $r_c^{-4} \alpha_{1D} N = 7.2 \times 10^4$ for the parameters of Figs. 3(b) and 3(c), respectively, in good agreement with the numerics.

The supersolid phase (SS in Fig. 3) retains the vortices. They are, however, pushed into the low-density regions and form a honeycomb vortex lattice that does not affect the structure of the supersolid droplet crystal (see Figs. 3(b) iii and 3(c) iii). At larger $r_c$, on the other hand, there is an interesting competition of length scales between the supersolid crystal and the vortex lattice. For sufficiently high rotation frequencies the vortex density and, hence, the vortex–vortex interaction, exceeds a critical value at which it becomes energetically favorable to form vortices inside the superfluid droplets [SSV in Fig. 3(c)]. As shown in Fig. 3(c) iv, the crystal structure of this additional vortex lattice is imposed by the triangular supersolid. The minimum rotation frequency, to support these states increases as $r_c$ is reduced. Since the confinement ceases for $\Omega \geq 1$, this implies a minimum interaction range $r_c^{\min}(\gamma N)$ in order to observe the SSV states for a given contact interaction $\gamma N$. For larger $r_c$ values, the on-site number of vortices successively increases, which eventually form a mesoscopic crystal of small vortex crystals as shown in Fig. 4.

In this work, we demonstrated novel vortex transitions in a realistic model of supersolids in the regime of weak interactions and large atom numbers per blockade sphere, which is most directly accessible to experiments. Extending the present study to the regime of strong coupling, future work may address how strongly correlated supersolids can be stabilized in finite-temperature cold atom experiments, which will more closely resemble the physical mechanism thought to underlie supersolidity in He. Along these lines, the realization of the found vortex crystals in the regime of strong interactions and small particle numbers will open the way towards studying quantum Hall-like phenomena in rapidly rotating self-assembled structures. This will require a beyond-MF description of the correlated many-body dynamics induced by rapid rotation.

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