Robustness of fractional quantum Hall states with dipolar atoms in artificial gauge fields

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The robustness of fractional quantum Hall states is measured as the energy gap separating the Laughlin ground state from excitations. Using thermodynamic approximations for the correlation functions of the Laughlin state and the quasihole state, we evaluate the gap in a two-dimensional system of dipolar atoms exposed to an artificial gauge field. For Abelian fields, our results agree well with the results of exact diagonalization for small systems but indicate that the large value of the gap predicted [Phys. Rev. Lett. **94**, 070404 (2005)] was overestimated. However, we are able to show that the small gap found in the Abelian scenario dramatically increases if we turn to non-Abelian fields squeezing the Landau levels.

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I. INTRODUCTION

Thirty years after its discovery in GaAs-AlGaAs heterojunctions [1], the fractional quantum Hall effect (FQHE) still remains the topic of current research, as nowadays this former solid-state phenomenon is rediscovered within the scope of quantum gases in two dimensions. Proposals to achieve the crucial ingredient, a perpendicular magnetic field, within such systems of neutral particles are at hand: Conceptually, the simplest of them is a rotation of the system [2,3], where the strength of the artificial magnetic field can be tuned by the frequency of rotation. However, addressing the regime where only the lowest Landau level (LLL) is occupied, and, at the same time, guaranteeing stability of the atomic cloud, requires a delicate balance between rotation and trap frequency, making an experimental realization of this proposal extremely hard.

An alternative way to circumvent this problem is to generate artificial fields by implementing a laser-assisted tunneling scheme within an optical lattice [4]. The idea is that during a hopping process stimulated by a laser field, the atom picks up the phase of the laser, which effectively simulates the action of a magnetic vector potential. In a very similar way, a U(1) Berry phase mimicking a gauge field can be inscribed into continuous systems via atom-laser coupling [5]. Furthermore, if the coupling involves more than two atomic states, it is possible to create a space-dependent degenerate subspace, which can be understood in terms of a non-Abelian Berry phase communicating between the degenerate atomic states. In this way, the atom-laser coupling scheme is generalized for synthesizing also non-Abelian gauge fields [6]. The same can be achieved in optical lattices if the laser-assisted tunneling is made sensitive on an additional, internal degree of freedom of the atoms [7]. These new possibilities have directed the attention to FQHE in such non-Abelian scenarios [8]. The practical feasibility of using lasers to implement artificial magnetic fields has been shown in pioneering experiments [9]. Furthermore, very recently, the implementation of spin-orbit coupling within a Bose-Einstein condensate can be considered an SU(2) gauge field realized in cold atoms [10].

The main motivation for seeking new realizations of the old fractional quantum Hall effect is the intriguing quasiparticles which occur as the excitations of fractional quantum Hall systems: Being neither bosons nor fermions, these so-called anyons behave exotically under interchange [11], as, instead of gaining simply a sign factor, a phase is obtained, which, in degenerate anyonic systems, may even be an element of some non-Abelian group. Due to this property, together with their topological and thus robust nature, quantum Hall states are especially interesting for quantum computation [12]. It is, therefore, most relevant to find fractional quantum Hall systems where a large energy gap separating the ground state from the excited states guarantees a high degree of robustness.

Considering dipolar atoms in a rotating trap, Ref. [3] claims to have achieved this. In this paper, we consider once again this scenario and find a much smaller gap. This finding directs our attention to the question regarding whether a robust FQHE might instead be realizable in a non-Abelian environment. From Refs. [13,14] it is known that in noninteracting systems the non-Abelian gauge field strongly influences the nature of the integer quantum Hall effect. By choosing an anisotropic field configuration it becomes possible to squeeze the Landau levels. Applying to such a squeezed scenario the thermodynamic approximation [15,16] used in the Abelian case in Ref. [3], we show in this paper that a dramatic increase of the energy gap can be achieved.

As in dipolar systems it turns out that the gap crucially depends on how it is defined, we first discuss this matter in Sec. II, which we conclude with a precise definition of the gap. Subsequently, we describe in Sec. III an analytic approximation allowing us to evaluate this gap in the thermodynamic limit. Section IV is dedicated to specifying the gauge potential. Here we show that Abelian and non-Abelian fields can be treated on the same footing. Finally, the concrete calculation and its results are described in Sec. V.

II. FRACTIONAL QUANTUM HALL EFFECT AND DEFINITION OF THE GAP

The physics of noninteracting particles confined to a plane under the influence of a perpendicular magnetic field is understood in terms of the quantized Landau levels. While this scenario may give rise to an integer quantum Hall effect, the FQHE requires repulsive interactions between the particles as its crucial ingredient. In the context of FQHE as a solid-state phenomenon, almost exclusively Coulomb interactions have been considered. For neutral fermions, however, it is most likely to have dipolar interactions. We, therefore, study the following Hamiltonian ($\hbar = 1$):

$$\hat{\mathrm{H}} = \sum_{j=1}^{N} \frac{1}{2m}[\boldsymbol{p}_j - \boldsymbol{A}(\boldsymbol{r}_j)]^2 + \sum_{j<k}^{N} V_{\mathrm{dd}}(\boldsymbol{r}_j,\boldsymbol{r}_k). \quad (1)$$

Here m is the mass of the particles and $V_{\mathrm{dd}}(\boldsymbol{r}_j,\boldsymbol{r}_k) = d^2/|\boldsymbol{r}_j - \boldsymbol{r}_k|^3$ the dipolar interaction between the particles with polarized dipole moments d. The gauge potential $\boldsymbol{A}$ is supposed to describe a constant gauge field perpendicular to the two-dimensional system and will be specified in Sec. IV.

It is known that, for practically any repulsive interaction, a very good trial wave function for the ground state is the so-called Laughlin function [17], which can be found by filling the lowest Landau level up to a filling factor ν [18]:

$$\Psi_{\mathrm{L}}(\{z_j\}) = \mathcal{N} \prod_{k<l}^{N} (z_k - z_l)^{1/\nu} \exp\left(- \sum_{i}^{N} |z_i|^2/4l_0^2 \right), \quad (2)$$

where $z_i \equiv x_i + iy_i$ are the positions of the particles, $\mathcal{N}$ is a normalization factor, and $l_0 \equiv \sqrt{1/B_0}$ is the magnetic length corresponding to a magnetic field of strength B_0. If $1/\nu$ is odd, the function is fully antisymmetric and thus describes fermionic systems, while for even $1/\nu$ it represents a bosonic wave function.

The Laughlin ground state can be considered a homogeneous liquid state. It is excited by piercing a hole into it. Choosing for simplicity the quasihole to be at the origin, the excited state is described by

$$\Psi_{\mathrm{qh}}(\{z_j\}) = \mathcal{N}' \prod_{j=1}^{N} z_j \Psi_{\mathrm{L}}, \quad (3)$$

with $\mathcal{N}'$ a normalization factor.

Note that, although having the same number of particles N as the Laughlin wave function (2), this wave function extends over a larger region S_{qh}^N than that for the Laughlin state, S_{L}^N. To see this we note that, assuming the thermodynamic limit $N \gg 1$, the density of the Laughlin state is uniform and equal to $n_{\mathrm{L}} = \nu/(2\pi l_0^2)$. The occupied region is therefore $S_{\mathrm{L}}^N = N/n_{\mathrm{L}}$. On the other hand, the density in the quasihole state (3) is $n_{\mathrm{qh}}(z) = n_{\mathrm{L}}[1 - \exp(-|z|^2/2)]$; see Ref. [16]. As a result, the difference in the number of particle for the states (2) and (3) in a finite area around the origin with the size much larger than l_0 is simply ν. These particles are pushed to the edge of the system such that the size of the quasihole state is

$$S_{\mathrm{qh}}^N = (N + \nu)/n_{\mathrm{L}}. \quad (4)$$

This consideration is relevant for the definition of the gap in a dipolar systems. An energy gap, vaguely defined as the energy difference separating the Laughlin state from the quasihole state, guarantees the robustness of the FQHE. But as one may think of different ways of creating a quasihole, it is important to be precise in the definition. We have to distinguish among three different ways to create quasiholes [16,19]: (i) by reducing the particle number, (ii) by increasing the area of the system at constant particle number, or (iii) by changing the magnetic field at constant particle number and constant area.

For electronic systems, in addition to the Coulomb repulsion between the electrons, a positively charged background (originated from ions) is present as a consequence of the electroneutrality of the system. The corresponding background potential stabilizes the Coulomb system, but it also adds to the energy of the quasihole, compensating the losses in the energy of the direct electron-electron Coulomb interaction due to lowering the number of electrons or their density. As a result, for any choice of the definition a positive energy gap is obtained [16,19].

Contrary to the electron case, dipolar systems have no such background potential, and the definition of the gap is crucial. As we will explicitly see in Sec. V, energy "gaps" defined according to (i) or (ii) have a negative sign. This would mean that the system is unstable against the creation of quasiholes. In fact, this, at first view, counterintuitive result is not astonishing: Since (i) and (ii) results in diluting the system and, therefore, in lowering the potential energy, it is obvious that in these cases we should find an energy gain. Therefore, the only meaningful definition for the energy gap in a dipolar system is according to (iii), where we compare the energy of a quasihole with the ground-state energy of the system with the same number of particles occupying the same area, similarly to the definition of the quasihole excitation energy in a normal Fermi system.

III. THERMODYNAMIC APPROXIMATION

In order to evaluate the gap, we follow an approach developed in Refs. [15,16]. Based on the plasma analogy [18], relating the physics of the Laughlin state to the one of a classical one-component plasma, analytic expressions for the correlation functions $g_0(z_1,z_2)$ and $g_{\mathrm{qh}}(z_1,z_2)$ of the Laughlin state and the quasihole state can be derived:

$$g_0(z_1,z_2) = \frac{\nu^2}{(2\pi)^2}\left(1 - e^{-\frac{|z_1-z_2|^2}{2}} - 2\sum_{j}^{\mathrm{odd}} \frac{C_j}{4^j j!} \times |z_1 - z_2|^{2j} e^{-\frac{|z_1-z_2|^2}{4}}\right), \quad (5)$$

$$g_{\mathrm{qh}}(z_1,z_2) = \frac{\nu^2}{(2\pi)^2}\left[\prod_{j=1}^{2}\left(1 - e^{-\frac{|z_j|^2}{2}}\right) - e^{-\frac{|z_1|^2+|z_2|^2}{2}} \times \left(\left|e^{\frac{z_1\bar{z}_2}{2}} - 1\right|^2 + 2\sum_{j}^{\mathrm{odd}} \frac{C_j}{4^j j!} \sum_{k=0}^{\infty} \frac{|F_{j,k}(z_1,z_2)|^2}{4^k k!}\right)\right], \quad (6)$$

$$F_{j,k}(z_1,z_2) = \frac{z_1 z_2}{2} \sum_{r=0}^{j} \sum_{s=0}^{k} \binom{j}{r}\binom{k}{s} \times \frac{(-1)^r z_1^{r+s} z_2^{j+k-(r+s)}}{\sqrt{(r+s+1)(j+k+1-(r+s))}}. \quad (7)$$

In these expressions for thermodynamically large systems edge effects are not considered. The quasihole state thus differs from the Laughlin state by the reduced particle density in the center, which according to the classification of the previous section yields a quasihole of type (i).

Given the expressions Eqs. (5)–(7), we may evaluate the energy difference Δ between the Laughlin and the quasihole state by solving the following integral:

$$\Delta = \frac{1}{2}\int d^2z_1 \int d^2z_2\, V_{\rm dd}(z_1,z_2)[g_{\rm qh}(z_1,z_2) - g_0(z_1,z_2)]. \tag{8}$$

Fortunately, we are able to derive a relation between the quantity from Eq. (8) and the gap related to a quasihole state according to (iii). First, we notice that $\Delta = E_{\rm qh}^{(N-\nu)} - E_0^{(N)}$, where these are the energies of $N-\nu$ particles in a quasihole state and N particles in the Laughlin state. We now may write $E_0 = N\epsilon_0$, where ϵ_0 is the energy of one particle in the Laughlin state. This quantity can readily be evaluated by substituting $z_- \equiv z_1 - z_2$ in $V_{\rm dd}(z_1,z_2) = V(z_-)$ and $z_- \equiv z_1 - z_2$ in $g_0(z_1,z_2) = g_0(z_-)$ and integrating:

$$\epsilon_0 = \frac{(2\pi)^2}{2\nu}\int_{-\infty}^{\infty} dz_-\, g_0(z_-)V_{\rm dd}(z_-). \tag{9}$$

Demanding a constant particle number, we may redefine the gap as

$$\Delta_N \equiv E_{\rm qh}^{(N-\nu)} - E_0^{(N-\nu)} = \Delta + \nu\epsilon_0. \tag{10}$$

This definition describes a quasihole created according to (ii). However, as argued in the previous section, apart from a fixed particle number, we should also demand a fixed area. For both states to occupy the same area, we thus have to modify the magnetic length l_0' of the excited state according to [see Eq. (4)]:

$$\frac{l_0'^2}{l_0^2} = \frac{N}{N+\nu}. \tag{11}$$

We now have to note that the energies in the dipolar system scale with l_0^{-3}. Since we wish to compare states at different magnetic fields, we define the gap at constant particle number and constant volume as:

$$\frac{\Delta_V}{l_0^3} = \frac{E_{\rm qh}^{(N)}}{l_0'^3} - \frac{E_0^{(N)}}{l_0^3}. \tag{12}$$

By noticing that

$$E_{\rm qh}^{(N)} = (\Delta + N\epsilon_0)\frac{N}{N-\nu}, \tag{13}$$

and approximating $N/(N-\nu) \approx (N+\nu)/N$ for large N, we find with Eq. (11):

$$\Delta_V = \Delta + \tfrac{5}{2}\nu\epsilon_0. \tag{14}$$

Before we calculate this quantity, we discuss different gauge potentials that might be realized.

IV. DIFFERENT GAUGE POTENTIALS

For the discussion of FQHE, it is feasible to choose a symmetric gauge for the gauge field. A perpendicular, magnetic field of strength B_0 is then described by the magnetic vector potential $\boldsymbol{A}_{\rm mag}(\boldsymbol{r}_j) = \frac{B_0}{2}(-y,x)$. In the context of artificial fields, it is possible to generalize this potential to be an element of SU(2). We choose it to have the form:

$$\boldsymbol{A}(\boldsymbol{r}_j) = \frac{B_0}{2}(-y,x) + (\alpha\sigma_y,\beta\sigma_x), \tag{15}$$

where α and β are additional, controllable parameters and $\sigma_{x,y}$ are Pauli matrices. Proposals to realize such gauge potentials have been made both for lattice systems [7] and for trapped gases coupled to laser fields [6]. Equation (15) contains the limit of a magnetic field, as can be seen by choosing $\alpha = \beta = 0$. For finite α and β, the potential (15) yields a constant non-Abelian gauge field perpendicular to the system.

To describe such a situation, we follow Ref. [13] and first consider the system without an Abelian flux, $B_0 = 0$. Diagonalizing the free part of the Hamiltonian, i.e., $(\boldsymbol{p} - \boldsymbol{A})^2$, one finds that the band structure of the free non-Abelian system contains Dirac cones, around which the sound velocity in the x direction, c_x, is proportional to α, while the sound velocity in the y direction, c_y, is proportional to β. Thus, the effective Hamiltonian for the low-energy physics has the form $H_{\rm D} = c_x\sigma_y p_x + c_y\sigma_x p_y$. Adding now the Abelian flux by replacing $\boldsymbol{p} \to \boldsymbol{p} + \frac{B_0}{2}(-y,x)$, we can bring this Dirac Hamiltonian to the form

$$H_{\rm D} = \begin{pmatrix} 0 & g_-a + g_+a^\dagger \\ g_-a^\dagger + g_+a & 0 \end{pmatrix}, \tag{16}$$

with $g_\pm = \sqrt{B_0/2}(c_y \pm c_x)$ and $a = \frac{1}{2}[\sqrt{B_0/2}(x - iy) + i\sqrt{2/B_0}(p_x - ip_y)]$. This Hamiltonian consists of an anti-Jaynes-Cummings term proportional to g_+ and a Jaynes-Cummings term proportional to g_-. The latter is zero in the isotropic limit $\alpha = \beta$, and then the spectrum consists of the usual relativistic Landau levels. As has been shown in Ref. [8], degeneracies between different Landau levels occur for $\alpha^2 \geqslant 3B_0$, giving rise to the possibility of a FQHE with non-Abelian anyons.

When $\alpha \neq \beta$, the situation is best described by introducing a squeezing parameter as done in Ref. [13],

$$\xi = -\tanh^{-1}\left(\frac{c_y - c_x}{c_y + c_x}\right). \tag{17}$$

The eigenstates are then obtained by a Bogoliubov squeezing transformation $S(\xi) = \exp[\frac{\xi}{2}(a^2 - a^{\dagger\,2})]$. For the lowest Landau level, corresponding to $E = 0$, the wave function $\Phi_{0,m}(z) = S(\xi)^\dagger[z^m\exp(-|z|^2/4l_0^2)]$ is found to be the eigenstate of the single-particle Hamiltonian of Eq. (16), and all particles have to be in the internal state $(0,1)^T$. Thus, if we restrict ourselves to the lowest Landau level, the two-component system becomes, effectively, a single-component system, and the only modification with respect to the Abelian case reduces to the action of the squeezing transformation $S(\xi)$, which consists just in the replacement of the original variable z by the squeezed one $\tilde{z}$:

$$[z \equiv x + iy] \to [\tilde{z}(\xi) \equiv \cosh\xi\; z - \sinh\xi\; \bar{z}]. \tag{18}$$

With this, we are able to treat both the Abelian scenario and the non-Abelian scenario on the same footing. As the Laughlin wave function is obtained by filling the lowest Landau level, the generalizations of the Laughlin wave function and the quasihole wave function, Eqs. (2) and (3), to the non-Abelian scenario are straightforwardly given by making the replacement Eq. (18). Accordingly, the correlation functions derived for the states in Eqs. (2) and (3) also hold for the corresponding squeezed states if we again make the substitution Eq. (18). The gap, as defined in Eq. (8) can then be evaluated by the integral

$$\Delta(\xi) = \frac{1}{2}\int d^2z_1 \int d^2z_2\; V_{\rm dd}(z_1,z_2) \\ [g_{\rm qh}(\tilde{z}_1(\xi),\tilde{z}_2(\xi)) - g_0(\tilde{z}_1(\xi),\tilde{z}_2(\xi))]. \qquad (19)$$

In the same way, we generalize the ground-state energy defined in Eq. (9) to be a function $\epsilon_0(\xi)$ of the squeezing. Following the derivation as in Sec. III, we finally arrive at the equation $\Delta_V(\xi) = \Delta(\xi) + \frac{5}{2}\nu\epsilon_0(\xi)$.

V. RESULTS

While in the Abelian scenario with $\xi = 0$ parts of the calculation can be done analytically, the squeezed scenario is more complicated as it demands a fully numerical treatment. However, since in both cases the steps of the calculation are the same, we describe in detail only the procedure for the Abelian case.

Before evaluating the integral, Eq. (8), we have to specify the coefficients C_j in Eqs. (5) and (6). It is shown in Ref. [15] that by setting all $C_j = 0$, a system with a completely filled Landau level is described, $\nu = 1$. For this choice of C_j, the resulting correlation functions are denoted by $g_0^{(1)}$ and $g_{\rm qh}^{(1)}$. In order to have a FQHE, we need a fractional filling, $\nu = 1/q$, which requires the coefficients C_j with $j \leqslant q$ to be nonzero. For fermions, the most robust effect is expected for $\nu = 1/3$, where the choice $C_1 = 1$ and $C_3 = -1/2$ is best suited. We call the corresponding correlation functions $g_0^{(3)}$ and $g_{\rm qh}^{(3)}$ and also define, for convenience, the differences $\Sigma_0 \equiv g_0^{(3)} - g_0^{(1)}$ and $\Sigma_{\rm qh} \equiv g_{\rm qh}^{(3)} - g_{\rm qh}^{(1)}$.

Turning now to the integral Eq. (8) with $\nu = 1/3$, we note that in the Abelian limit it reduces to the one considered in Ref. [3]. As our numerical result, however, drastically differs from Ref. [3], a careful analysis is in order. Therefore, we split the integral Eq. (8) into two parts, $P_1 \equiv \int dz_1 \int dz_2\, V_{\rm dd}(g_{\rm qh}^{(1)} - g_0^{(1)})$, which is analytically solvable, and $P_2 \equiv \int dz_1 \int dz_2\, V_{\rm dd}\, (\Sigma_{\rm qh} - \Sigma_0)$, which we treat numerically.

For the analytic part we find $P_1 = -\sqrt{2\pi}/\nu^2 \frac{d^2}{l_0^3}$. Note that for $\nu = 1$, this negative number would be the full, completely analytic result for the energy difference Δ defined in Eq. (8). This clearly shows what we have anticipated in Sec. II, namely that this definition is not the appropriate one for the energy gap in a dipolar system.

Before we evaluate P_2 numerically, we examine the asymptotic behavior of the integrand. As the divergence in the interaction term for $z_1 \to z_2$ is compensated by the vanishing of the correlations, this limit can easily be handled by a regularization of the integral. The limit of $z_+ \equiv z_1 + z_2 \to \infty$, however, turns out to be problematic: For finite particle distance, $|z_1 - z_2| < \infty$, this contribution is not suppressed by the interaction, and the convergence of the integral Eq. (8) requires that Σ_0 and $\Sigma_{\rm qh}$ have the same asymptotic behavior. However, the completely different structure of both functions obscure the latter. Contrariwise, we should note that if we truncate the infinite sum in $\Sigma_{\rm qh}$, stemming from the s sum in Eq. (7), this expression gets exponentially damped for large center-of-mass coordinates, while Σ_0 depends only on the relative coordinates, yielding $\Sigma_0 - \Sigma_{\rm qh} \neq 0$ for $|z_+| \to \infty$.

To circumvent this problem, we bring Σ_0 to a form similar as $\Sigma_{\rm qh}$, which is possible by factoring out a damping $\exp[-(|z_1|^2 + |z_2|^2)/2]$ and Taylor expanding the remaining exponential $\exp[|z_+|^2]$. We are then able to write

$$\Sigma_0(z_1,z_2) = e^{-\frac{|z_1|^2+|z_2|^2}{2}} \sum_j \frac{-2C_j}{4^j j!} \sum_{k=0}^{\infty} \frac{\left|F_{j,k}^{(0)}(z_1,z_2)\right|^2}{4^k k!}, \qquad (20)$$

$$F_{j,k}^{(0)}(z_1,z_2) = \sum_{r=0}^{k}\sum_{s=0}^{k} \binom{j}{r}\binom{k}{s} (-1)^{j-r} z_1^{r+s} z_2^{j+k-(r+s)}. \qquad (21)$$

As now each term in both Σ_0 and $\Sigma_{\rm qh}$ is damped by a factor $\exp[-(|z_1|^2 + |z_2|^2)/2]$, they all vanish in the limit $|z_+| \to \infty$, and we may truncate the infinite sums at a sufficiently large value of k. Note that due to the different orders in z_1 and z_2 of $F_{j,k}$ in Eq. (7) and $F_{j,k}^{(0)}$ in Eq. (20), the sum in Σ_0 should contain two more terms than the sum in $\Sigma_{\rm qh}$ for a quick convergence.

Now we are able to perform the numerical integration. The error due to the truncation still is 5% for 10 terms but can be minimized by a finite-size analysis of our results taking into account up to 25 terms. We then find $P_2 = (0.1875 \pm 0.0010)\frac{d^2}{l_0^3}$, where the numerical error has been approximated by the deviation from the smooth fit in Fig. 1. With this, we find $\Delta = 0.5(P_1 + P_2) = -(0.0455 \pm 0.0010)\frac{d^2}{l_0^3}$. As we have argued in Sec. II, this negative value is due to the reduced density of the system.

We continue with calculating the gap as defined in Eq. (10). Therefore, we have to evaluate the integral Eq. (9), which in the isotropic case, $\xi = 0$, reads $\epsilon_0 = \frac{\sqrt{\pi}}{2\nu}(\frac{\sqrt{2}}{2} - \frac{15}{32})\frac{d^2}{l_0^3}$, from which we find that also $\Delta_N < 0$. The negative value for Δ_N can be understood by noticing that as the particle taken away at the

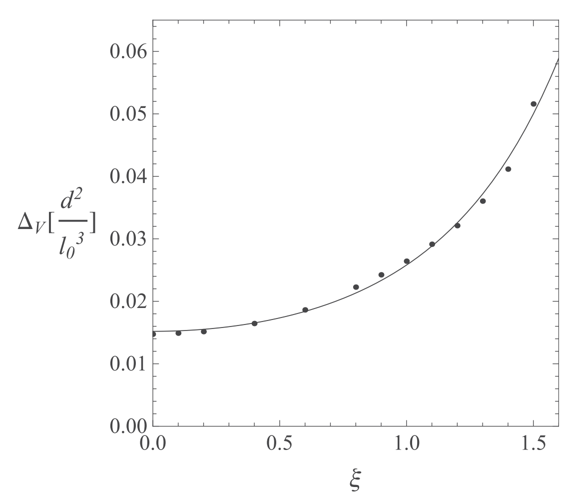

FIG. 1. The gap Δ_V at constant volume and constant particle number as a function of the squeezing parameter ξ: The dots are obtained by a numerical evaluation of Eq. (8) for different ξ, a fit of this data yields the solid line.

origin now has been added at the edge of the system increasing its volume, so Δ_N corresponds to the energy of a quasihole according to (ii). As long as such a process is possible, the system is unstable as it tries to reduce its density by diluting.

Finally, we turn to definition Eq. (14). Only in this case, we obtain a positive gap, $\Delta_V = (0.0132 \pm 0.0020)\frac{d^2}{l_0^3}$, which, however, is much smaller than the number found in Ref. [3], $(0.9271 \pm 0.019)\frac{d^2}{l_0^3}$, but compares well with the gap obtained via exact diagonalization of a small dipolar system in Ref. [20], where the discrepancy to Ref. [3] has been attributed to the different system size. The deviation of our result from the one obtained in Ref. [3] is due to the different handling of the infinite sum in Eq. (7). While by the method described here, based on the expansion of Eq. (5) into Eq. (20), the correct asymptotic behavior of $g_0 - g_{\mathrm{qh}}$ is guaranteed for arbitrarily large variables z_1 and z_2 even after truncating the sum, the opposite is true if one just truncates the sum in Eq. (7) and leaves g_0 as defined in Eq. (5). As argued above, the latter always produces wrong results for large center-of-mass coordinates, such that the validity of the integrand in Eq. (8) is restricted to a finite regime around the hole. It turns out that a very large number of terms would have to be taken into account, in order to capture the whole regime which substantially contributes to the integral, and distinguish it from the regime where contributions are solely due to the truncation.

Turning now to the non-Abelian case, we may repeat the same procedure for finite squeezing ξ, but now we have to evaluate $\Delta(\xi)$ as defined in Eq. (19). Here, the whole integral has to be solved numerically. Again, we find negative values for $\Delta(\xi)$ and $\Delta_N(\xi)$, which decrease with larger ξ. However, as the ground-state energy $\epsilon_0(\xi)$ also increases with ξ, the gap $\Delta_V(\xi)$ at constant particle number and constant volume finally has a positive balance for all ξ. As shown in Fig. 1, it increases with ξ, and a convenient fit to the numerical data is found to be:

$$\Delta_V(\xi) = \Delta_V(0)\exp(\alpha\,\xi^2). \tag{22}$$

We obtain $\alpha = 0.529$ and $\Delta_V(0) = 0.0152\ d^2/l_0^3$.

To understand this behavior, we note that the squeezing allows the particles to get closer in one direction, while the particle distance is increased in the other direction. Due to $1/r^3$ behavior of the dipole-dipole interaction, the interaction energy is much more sensitive to changes of the density distribution at short distances rather than at large ones. Thus, compressing in one direction and stretching in another one increases the interaction energy. As a consequence of Eq. (14), this gives rise to a bigger energy gap.

VI. CONCLUSION

Summarizing this work, we have shown that, dramatically differing from the predictions in Ref. [3], only a small energy gap separates the Laughlin state from quasihole excitations in systems of dipolar quantum gases with artificial magnetic fields. However, by considering scenarios where a non-Abelian gauge field introduces an anisotropy into the system, an exponential increase of the gap can be achieved and may allow for robust fractional quantum Hall states.

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