Collective Light Emission Revisited: Reservoir Induced Coherence

Yumian Su,1,2,* Dieter Bimberg,1 and Andreas Knorr3

1Institut für Festkörperphysik, Technische Universität Berlin, Hardenbergstraße 36, 10623 Berlin, Germany
2DSO National Laboratories, 20 Science Park Drive, Singapore 118230, Singapore
3Institut für Theoretische Physik, Nichtlineare Optik und Quantenelektronik, Technische Universität Berlin, Hardenbergstraße 36, 10623 Berlin, Germany

Alexander Carmele

Institute for Quantum Optics and Quantum Information, 6020 Innsbruck, Austria

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We study the collective emission of a few emitters (up to three) in a cavity. In addition to the radiation coupling responsible for sub- and superradiance, we investigate emitters additionally coupled through a joint carrier reservoir. For such emitters, typically embedded in a solid state environment, the carrier reservoir provides a continuous pumping mechanism for the steady state emission. We show that the statistical properties of the emitted light depend strongly on the interaction between the emitters and the reservoir. Unexpectedly, the presence of the reservoir enhances the coherence of the emitted light already for a few emitters. This results from the fact that the carrier reservoir introduces new many-body correlations to the electronic transition and in this way suppresses multiphoton processes.

Introduction.—The case of the collective spontaneous emission of $N$ identical atoms was first studied by Dicke in 1954 [1]. The atoms are placed closer than the emission wavelength and thus are efficiently coupled through the radiation field without retardation effects. The corresponding emission can be superradiant (subradiant) where spontaneous emission is enhanced (suppressed) due to constructive (destructive) interaction of the emitters. This phenomenon of collective emission has since been studied intensively both in theory and in experiment [2–13]: Superradiance has been achieved in the transient state in atomic [2], quantum dot [14,15], and quantum well [16] systems, and also with atoms in high-finesse cavities [17] to circumvent coherence loss due to energy shifts [18]. Subradiance is achieved in experiments for two ions [19], a diatomic molecule in an optical lattice [20], and a collection of atoms [21].

In contrast to well isolated atoms, a solid state environment for the emitters gives rise to properties that are different or absent in atomic systems. For example, it has been observed experimentally that semiconductor quantum dots decay nonexponentially [22,23]. It has also been proved theoretically that solid state specific effects, such as many-body correlation in a quantum dot, play a significant role for the light statistics in one-emitter-cavity single photon sources [24,25].

To our knowledge, the role of a charge carrier reservoir which provides an electron and hole supply for the cooperative emission of color centers, quantum dots, impurities in diamonds, or other single solid state emitters, has not been studied before. In this Letter, we extend the work of the atomic model on the collective emission of several atoms [9,10,17,26] to that of several solid state emitters by including additional contributions from many-body terms in the presence of charge carrier reservoirs. To investigate the effect of a reservoir on the emitted light statistics, we focus on the photon-photon correlation function $g^{(2)}(0)$ as a measure of coherence of the emitted radiation. In contrast to the general assumption that a reservoir usually introduces decoherence, we show that the carrier reservoir plays a crucial role in enhancing the coherence in a few emitter system. These theoretical predictions might trigger yet unavailable experiments.

Model.—Our system consists of $N$ identical emitters placed in a cavity [18]; each emitter has two energy levels (Fig. 1). In contrast to isolated atomic two-level systems, in solid state emitters, carriers are scattered into (out of) the emitter states through a joint carrier reservoir (CR). We compare two models. (a) A one-electron atomic model without a reservoir, where carrier density at the upper and lower levels is fully correlated. That is, there is just a single electron which is always present in one emitter (closed system). (b) A solid state model, where carrier density at the upper and lower levels is independent since the levels are coupled to an external CR (open system).

![FIG. 1 (color online). Identical two-level emitters in cavity are compared for two cases: (a) atomic model where electron and hole densities are equal, and (b) a solid state model where a CR allows their mutual independence.](image)
To characterize the intensity and the coherence properties of the emitted light, observables of interest are the average photon number in the cavity $\langle c \dagger c \rangle$ and the second-order photon correlation function $g^{(2)}(0) = \langle c \dagger c c \dagger c c \rangle / \langle c \dagger c \rangle^2$. Here, $c \dagger$ and $c$ are cavity photon creation and annihilation operators, respectively. The expectation value is denoted by $\langle \cdots \rangle = tr(\cdots \rho)$, with $\rho$ being the statistical operator. In a Fock basis, the average photon number can be written as $\langle c \dagger c \rangle = \sum_{n=1}^{\infty} n p_n$, and the two-photon correlation as $\langle c \dagger c c \dagger c c \rangle = \sum_{n=2}^{\infty} n(n-1) p_n$ [27]. Here, $p_n = \langle \langle n|n \rangle \rangle$, is the probability of finding $n$ photons in cavity. Therefore, to study the properties of cavity photons, we need to derive the equation of motion for $p_n$.

We use the system Hamiltonian: $H = H_0 + H_{\text{el-ph}} + H_{\text{el-cr}}$, where $H_0$ is the free Hamiltonian of the cavity photons (mode energy $\hbar \omega$) and electrons: $H_0 = \hbar \omega c \dagger c + \hbar e_v \sum_{n=1}^{N_v} a_1^{\dagger} a_i + \hbar e_c \sum_{n=1}^{N_c} a_\nu^{\dagger} a_i$, where $N_v$ is the number of emitters in the cavity, which is in resonance with the electron transition ($\omega = \varepsilon_c - \varepsilon_p$), $\hbar e_c$ is the energy of electrons at the upper level (or conduction band state), and $\hbar e_v$ is the energy of electrons at the lower level (or valence band state). $a_1^{\dagger}$ ($a_1$) creates an electron at conduction (valence) band state in the $i$th emitter. $H_{\text{el-ph}}$ describes the interaction between electrons and cavity photons: $H_{\text{el-ph}} = -\hbar \sum_{n=1}^{N_v} (g a_1^{\dagger} a_i c \dagger + g^* a_1 a_i^\nu c)$, where $g$ is the coupling constant between cavity photons and the emitter electronic transition.

$H_{\text{el-cr}}$ describes the interaction between electrons in the emitters and the CR. The coupling of the active levels to the CR can be through carrier-carrier or/and carrier-phonon scattering, which pump the collective emission. As a typical example we apply the carrier-phonon scattering [28] to derive effective scattering rates. Our general results remain valid for other emitter-reservoir couplings, independent of the specific type of coupling, as long as they can be treated in the Born-Markov approximation.

To include the cavity loss and pure dephasing present in our description, we use the Lindblad form, $\dot{L}_C(\rho) = \kappa(2c_p c^\dagger - c^\dagger c \rho - \rho c c^\dagger)$ and $\dot{L}_P(\rho) = \gamma/4 \sum_{n=1}^{N_v} [2\sigma_i^\nu \rho \sigma_i^\nu - \sigma_i^\nu \sigma_i^\nu - \rho \sigma_i^\nu \sigma_i^\nu - \sigma_i^\nu \sigma_i^\nu |a_1 a_1^{\dagger} a_1^{\dagger} a_1^{\dagger} a_1^{\dagger}}, respectively.

Dynamical equations.—We derive equations for observables by using the Heisenberg equation of motion. As in Ref. [17], we assume that the emitters are identical and obey the same initial conditions. Therefore, the expectation values of the observables of the $i$th emitter equal those of the 1st emitter.

The photon probability function $p_n$ evolves according to

$$\frac{dp_n}{dt} = -2\sqrt{n} N \text{Im}(g|\langle n+1 | a_1^{\dagger} a_1^\nu \rangle - \kappa (2n+1) |\langle n+1 | n a_1^{\dagger} a_1^\nu \rangle + 2\kappa \sqrt{n+1} |n+2|\langle n+2 | n+1 a_1^{\dagger} a_1^\nu \rangle - P \langle n+1 | n a_1^{\dagger} a_1^\nu \rangle + \mathcal{F}. \tag{2}$$

Here, $\gamma$ is the pure-dephasing rate. $P$ is the pump rate derived from carrier-CR interaction. (A detailed form of $P$ is provided in the Supplemental Material [29].) $\mathcal{F}$ represents the coupling of the photon-assisted polarization to higher order correlations due to the electron-photon interaction:

$$\mathcal{F} = ig^* \sqrt{n+1} |\langle n+1 | n a_1^{\dagger} a_1^\nu \rangle - \langle n | n a_1^{\dagger} a_1^\nu \rangle - \langle n+1 | (n+1) a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu \rangle + \langle n | n a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu a_1^\nu \rangle | - ig^* \sqrt{n+1} |(N-1) \times |\langle n+1 | n a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu a_1^\nu \rangle - \langle n | n a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu a_1^\nu \rangle | + ig \sqrt{n+2} |(N-1) |\langle n+2 | n a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu a_1^\nu \rangle - \langle n | n-1 | n-1 a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu a_1^\nu \rangle |. \tag{3}$$

Here, on the right-hand side, the 3rd and 4th terms describe the many-body interaction within the 1st emitter; the 5th to 8th terms describe the cross coupling between different emitters. From Eqs. (1) to (3) it can be recognized that $p_n$ is driven by $\langle n+1 | n a_1^{\dagger} a_1^\nu \rangle$, which in turn is driven by four-operator terms such as $\langle n+2 | \times | n | a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu a_1^\nu \rangle$. The appearing hierarchy is illustrated in Fig. 2. Now, this hierarchy can be addressed in two ways: (a) treating isolated emitters with fixed electron number (atomic model), and (b) open system emitters with an electron occupation influenced by the external solid state carrier reservoir (solid state model). Next, we specify the theory for these two important cases.

(a) Atomic model: For one electron per emitter, the electron density $\langle a_1^{\dagger} a_1 \rangle$ and hole density $\langle 1 - (a_1^{\dagger} a_1) \rangle$ in the same emitter are fully correlated, namely, the electron density equals the hole density. In this case, the two-electron interaction term, $\langle a_1^{\dagger} a_1^{\dagger} a_1^\nu a_1^\nu \rangle$, in Eq. (3) is zero as there is only one electron per emitter. Equation (3) also depends on four-operator terms: $a_1^{\dagger} a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu a_1^\nu$ and $a_1^{\dagger} a_1^{\dagger} a_1^\nu a_1^\nu a_1^\nu a_1^\nu$. 

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backbone of our reported results, below. Completely new scattering paths are included and form the pumping event. In contrast to Lindblad approaches [30], accounting for non-Markovian contributions in the mechanism stems from a many-body carrier reservoir-theoretical emission (CRACE). Note, our derived incoherent pump state model is now called carrier-reservoir-assisted collective emission (CRACE). The collective emission calculated from this solid state model (CRACE), not in the atomic model (ACE).

For $N$ emitters, the emitter reservoir, in the solid state model, electron and hole densities are independent. Then the circled terms in Fig. 2 occur and they enter the system dynamics. This leads to additional coupling paths among emitters via the joint reservoir. Including equations of motion for the four- and six-operator terms, for $N = 2$, the solid state model consists of a closed set of 16 equations to fully describe the system dynamics. These 16 observables are shown in Fig. 2, excluding the circled terms. When $N = 3$, system dynamics are determined by 24 equations (Supplemental Material [29]). The collective emission calculated from this atomic model is now called atomic collective emission (ACE).

(b) Solid state model: Because of their coupling to a reservoir, in the solid state model, electron and hole densities are independent. Then the circled terms in Fig. 2 occur and they enter the system dynamics. This leads to additional coupling paths among emitters via the joint reservoir. Including equations of motion for the four- and six-operator terms, for $N = 2$, the solid state model consists of a closed set of 16 equations to fully describe the system dynamics. These 16 observables are shown in Fig. 2. For $N = 3$, we need 40 equations (Supplemental Material [29]). The collective emission calculated from this solid state model is now called carrier-reservoir-assisted collective emission (CRACE). Note, our derived incoherent pump mechanism stems from a many-body carrier reservoir theory, accounting for non-Markovian contributions in the pumping event. In contrast to Lindblad approaches [30], completely new scattering paths are included and form the backbone of our reported results, below.

Results in the stationary limit. —When $N = 1$, our set of ACE equations describe a one-atom laser [31], and the set of CRACE equations describe a single externally pumped emitter. In particular, the photon number and statistics agree well with previous work [25]. When $N > 1$, there are three differences between ACE and CRACE in the stationary limit:

(i) Reservoir-induced additional quantum paths lead to less radiation suppression in CRACE: When $N = 3$, the photon number as a function of pump rate $P$ for both ACE and CRACE are shown in Fig. 3, comparing collective emission vs independent emission (emitters are not coupled through radiation). For both ACE and CRACE, as the effective pump rate increases, two regimes appear: a subradiant regime where spontaneous emission is suppressed due to destructive interaction of the emitters (low pump regime, shaded) and a superradiant regime where spontaneous emission is enhanced due to constructive interaction of the emitters (high pump regime). Note that for $N = 3$, the difference in the photon number for collective and independent emission is small, as already shown by exact results for atomic models [1,17]. It can also be clearly recognized that the subradiant regime in CRACE is less pronounced in comparison with that in ACE. Even if this is a small effect, it is central to understand the drastic difference in the light statistics discussed in points (ii) and (iii) below.

In general, for low pumping, on average, some emitters are in the excited states and many more are in the ground states. For two emitters, an antisymmetric dark state forms, which contains one emitter in the excited state and the other in the ground state. This dark state does not radiate, and is referred to as a subradiant state [1]. For many emitters, the subradiant state can be considered as an ensemble of pairs of emitters in antisymmetric states [21]. In contrast to ACE, in CRACE, the carrier reservoir induces additional coupling paths between the subradiant state to superradiant states, enabling the subradiant state to contribute to the light emission. This results in the reduced radiation suppression.

(ii) Reservoir-induced additional quantum paths lead to monotonic behavior of $g^{(2)}(0)$ as a function of $N$ in CRACE: As the presence of the carrier reservoir in CRACE reduces the magnitude of destructive interaction among emitters in the low pump regime, we focus on this regime and discuss the photon-photon correlation $g^{(2)}(0)$ for ACE and CRACE. If $g^{(2)}(0) = 1$, this indicates a coherent (laserlike) emission; $g^{(2)}(0) = 0$ represents a single photon emission event (photon antibunching), and $g^{(2)}(0) > 1$ photon bunching, i.e., more chaotic radiation.
Ref. [9]. In ACE, when $g$ or nonmonotonic monotonically in CRACE but not in ACE (Fig. 4). This nonmonotonic $g^{(2)}(0)$ in ACE was already reported in Ref. [9]. In ACE, when $N = 2$, low pumping takes one emitter to the excited state $|e\rangle$ while the other remains in the ground state $|g\rangle$. This results in the formation of a bright symmetric state $(|eg\rangle + |ge\rangle)$ and a dark antisymmetric state $(|eg\rangle - |ge\rangle)$. The bright state can emit a single photon, but the dark state does not radiate. However, the dark state can be electrically pumped by the carrier reservoir. The next incoherent pump event takes the dark reservoir to the bright state $|ee\rangle$, resulting in the emission of two photons, which increases the probability of bunching. When $N = 3$, low pumping can only promote one emitter to the excited state and the other two remain in the ground state, which again forms a bright and two dark states. In the next pump event, one of the dark states can either be promoted to a superradiant state, which emits a photon pair, or to a state, which emits a photon but ends in a one-excitation dark state, again. The former contributes to bunching, the latter to antibunching by a single-photon emission. Therefore, photons in $N = 3$ case are more antibunched than those in $N = 2$, or $g^{(2)}(0)|_{N=3} < g^{(2)}(0)|_{N=2}$.

In CRACE, however, the dark superradiant state can couple to superradiant states through the joint reservoir, making it possible to radiate. Therefore, it is less probable in CRACE for this state to be promoted to a superradiant state in the next pump event, in comparison to ACE. As either a photon pair or a single photon can be emitted, depending on the nature of this superradiant state, multiphoton events are suppressed in CRACE. When $N$ increases from 2 to 3, the extra single photon event is suppressed, or the contribution to photon antibunching is suppressed. This leads to a monotonic increase in $g^{(2)}(0)$ in CRACE.

(iii) Reservoir-induced multiphoton suppression leads to more coherent light emission in CRACE: In the low pump regime, Fig. 4 shows that for $N = 2$ and 3, the emitted photons are coherent in CRACE [$g^{(2)}(0) = 1$] in comparison to ACE which is chaotic [$g^{(2)}(0) > 1$]. In CRACE, carrier reservoir induces additional coupling paths between the subradiant state to superradiant states. This enables the subradiant state to radiate and makes multiphoton events less probable in subsequent pump events. Clearly, multiphoton events are suppressed: For example, when $N = 3$, the photon probabilities yield: $p_{\text{ph}}^{\text{CRACE}} = 92\%$, $p_{\text{ph}}^{\text{ACE}} = 44\%$, $p_{\text{ph}}^{\text{CRACE}} = 9\%$, and $p_{\text{ph}}^{\text{CRACE}} = 0.9\%$. As $g^{(2)}(0) = \sum_{n=2}^{\infty} n(n-1)p_n/\sum_{n=1}^{\infty} np_n$, smaller multiphoton probabilities lead to a smaller $g^{(2)}(0)$ in CRACE.

The strong difference in the coherence properties of the emitted light, manifested in the $g^{(2)}(0)$ function between ACE and CRACE, also persists for a broad range of cavity loss $\kappa$, as shown in Fig. 5. Here, the difference is even more drastic: In particular for intermediate losses, where the atomic emission is chaotic, the coherence persists through the reservoir-emitter interaction. When cavity loss becomes very big, both ACE and CRACE are in theDicke superradiant regime and coherence is naturally restored.

Note that if the emitters are detuned symmetrically (equally in one direction) from the cavity resonance, the above analysis is still valid. For example, for a detuning of 30 $\mu$eV, the $g^{(2)}(0)$ function decreases less than 1% in ACE and remains roughly unchanged in CRACE. If two emitters are detuned from the cavity mode asymmetrically, the change in $g^{(2)}(0)$ is larger. For example, with one emitter on resonance with the cavity mode and the other detuned by 30 $\mu$eV, the change in $g^{(2)}(0)$ can be as large as 20% in ACE and 6% in CRACE. Quantitatively, with this change, the emitted light is still more coherent in CRACE than in ACE.

**Conclusions.**—We generalized the description of collective few-emitter emission in the presence of a joint carrier reservoir. In the subradiant regime, the reservoir induces additional quantum paths among the emitters, resulting in less radiation suppression and monotonic behavior of the $g^{(2)}(0)$ function. However, most importantly, the reservoir also suppresses multiphoton events, drastically changing...
the photon statistics to be coherent in contrast to chaotic emission without a reservoir. We believe that the observed difference in the emission will be of large importance for the understanding of solid state quantum optics and will trigger important experimental work.

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*yumiansu@sol.physik.tu-berlin.de