Optical Feedback-Enhanced Photon Entanglement from a Biexciton Cascade

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In this Letter we present a novel approach by using a quantum feedback scheme [31] in order to control and enhance the photon pair entanglement in a biexciton cascade with non-negligible fine-structure splitting. Many feedback control schemes include a measurement process that would be detrimental for our aim to improve entanglement since measurements usually destroy quantum coherence through the collapse of the wave function. Our feedback scheme, however, fully preserves quantum coherence.

Introduction.—Entanglement is one of the most intriguing aspects of quantum mechanics since it creates statistical correlations which appear paradoxical from a classical point of view [1–4]. Furthermore, it is crucial for applications of quantum mechanics, such as quantum cryptography [5,6] or quantum computing [7]. One prominent source of entangled photons is the biexciton cascade in a quantum dot (QD) [8–17]. A biexciton can decay through two different decay paths by the emission of two photons (cf. Fig. 1). Due to the conservation of angular momentum, the photons are correlated in their polarization, both being either horizontally or vertically polarized. In an ideal case, the two decay paths are otherwise indistinguishable, rendering the two photons polarization entangled.

However, many factors may diminish the achievable photon entanglement, one of the most prominent being the excitonic fine-structure splitting [20,21]. This effect results in slightly different energies of the two intermediate exciton levels (cf. Fig. 1), which makes the two decay paths distinguishable by the chronology of the photon energies and therefore reduces their entanglement.

Several strategies have been proposed to overcome this problem, for example changing the internal properties of the QD [16,22,23], applying external fields [12,24] or strain [25], using a high-quality cavity [13,15,26], spectral [11] or temporal filtering [27], or even changing the time order of the emitted photons [28–30]. However, all of these strategies involve complex changes in the nanostructure, extensive additional external components, making it hard to retrieve the photons or block off a large proportion of them.

In this Letter we present a novel approach by using a coherent quantum feedback scheme [31] in order to control and enhance the photon pair entanglement in a biexciton cascade with non-negligible fine-structure splitting. Many feedback control schemes include a measurement process...
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\hat{H} = \hbar \int_{-\infty}^{\infty} dk \gamma(k) \left( e^{i(\omega_k - \omega_0 - \delta/2)t} \hat{b}^\dagger \hat{c}_{H,k} + e^{i(\omega_k - \omega_0 - \delta/2)t} \hat{b} \hat{c}_{V,k} \right) + e^{i(\omega_k - \omega_0 - \delta/2)t} \hat{c}_{H,k} \hat{c}_{V,k} + e^{i(\omega_k - \omega_0 + \delta/2)t} \hat{c}_{H,k} \hat{c}_{V,k} + e^{-i(\omega_k - \omega_0 + \delta/2)t} \hat{c}_{H,k} \hat{c}_{V,k} \right) + \text{H.c.}
\]  

(1)

\( \hat{b}, \hat{c}_{H/V}, \) and \( \hat{g} \) are the annihilation operators of the biexciton state \( |B\rangle \), the two exciton states \( |X_{H/V}\rangle \), and the ground state \( |G\rangle \), respectively (see Fig. 1). The two exciton states are differentiated by the polarization \((H/V)\) of the photons to which they couple. \( \hat{c}_{H,k} (\hat{c}_{V,k}) \) is the annihilation operator of a photon with wave vector \( k \) and horizontal (vertical) polarization. The dagger symbol (\( \dagger \)) marks the corresponding creation operators. We use the free space dispersion relation \( \omega_k = c|k| \). \( \gamma(k) \) is the wave-vector-dependent light-matter coupling constant. We assume an equal coupling to the electric field for the biexciton-exciton and the exciton-ground state transitions. Without feedback, \( \gamma(k) \) can be considered constant over the frequency range of the transitions: \( \gamma(k) = \gamma_0 / \sqrt{2} \). We introduce feedback with a delay time \( \tau_f \) by setting \( \gamma(k) = \gamma_0 \sin(kL), \) \( L = \tau_f c/2 \), fulfilling the boundary condition for electric fields at a mirror surface at distance \( L \) [19,40,41] (see Fig. 1). This effective 1D model is closest to a finite waveguide; however, the similarity to other implementations of a defined surface at distance \( \lambda_0 = 942 \) nm. For simplicity, we put the biexciton binding energy \( \beta = 0 \), which is experimentally feasible [52], however not fundamentally necessary for our proposal to work. We checked that for \( \beta \neq 0 \) the concurrence could also be enhanced by introducing time-delayed feedback.

When the QD has decayed to its unique ground state and the photonic wave packet has traveled away from it, i.e., in the limit of \( t \to \infty \), the overlap \( \zeta(t) \equiv C \) is a good measure for the photon entanglement [11,39,51]. In bipartite quantum systems, \( C \) is referred to as the concurrence.

For our simulations, we use \( \omega_K = 2 \text{ fs}^{-1} \), which corresponds to photon energies of about 1.3 eV and a wavelength of \( \lambda_0 = 942 \) nm. For simplicity, we put the biexciton binding energy \( \beta = 0 \), which is experimentally feasible [52], however not fundamentally necessary for our proposal to work. We checked that for \( \beta \neq 0 \) the concurrence could also be enhanced by introducing time-delayed feedback.

We set \( \gamma_0 \) such that it leads to an intrinsic biexciton decay time \( \tau_B \) of about 300 ps. For the photon field, we use 8000 modes with frequencies distributed equally around \( \omega_K \) over a range of 0.4 ps\(^{-1}\).

The following discussion is structured as follows: We first show the results for small feedback delay times \( \tau_f \) and afterwards discuss larger delay times. In each case we present time traces of important dynamical variables (i.e., the biexciton probability \( |b(t)|^2 \) and the two-photon wave-function overlap \( \zeta(t) \), see Fig. 2) and explain them with the help of the respective photon number distribution (Fig. 3).

Results: (a) small delay.—In the short feedback regime the feedback time \( \tau_f \) is considerably smaller than the biexciton lifetime \( \tau_B \). We therefore choose the mirror distance \( L \) between \( 10^4 \lambda_0 \) and \( 10^3 \lambda_0 + (\lambda_0 / 4) \), corresponding to the millimeter range and a delay of \( \tau_f \approx 63 \) ps. For the used distance range, the feedback phase \( \varphi_K \) varies between \( \pi \) and 0.

Figures 2(a) and 2(b) compare the time dynamics with and without the mirror. As depicted in Fig. 2(a), the biexcitonic decay is strongly modified after the time \( t = \tau_f \approx 63 \) ps. Depending on the phase of the feedback, we either get a higher (\( \varphi = 0 \)) or lower (\( \varphi = \pi \)) decay rate. This phenomenon can be explained by a modification of the local density of photonic states at the position of the QD by the mirror, or, equivalently, by a phase-dependent driving of the excitonic transitions by the feedback field. This effect is expectably only “noticed” by the system after \( \tau_f \). Despite the feedback, the electronic excitations fully decay, since only photons emitted into one half-space get reflected while
they experience no boundary in the other half-space and travel away from the QD. The slowest decay is observed for $\phi_X = \pi$ [Fig. 2(a), blue dotted line], since the QD is located in a field node and its decay is strongly decelerated.

The two-photon wave-function overlap $\zeta(t)$ [Fig. 2(b)] is also strongly altered by optical feedback. For feedback phases of about $\phi_X = 0$, we see an increase after $t_f$, which for $t \to \infty$ leads to a substantially higher concurrence than in the case without feedback. We will first focus on this effect and discuss the case of $\phi_X = \pi$ later. To explain the increased entanglement for $\phi = 0$, we study the photon number $\hat{c}^\dagger_k \hat{c}_k$ in the limit of $t \to \infty$ (Fig. 3). The photon number distribution represents the energy density spectrum resolved in frequency. Figure 3(a) shows that the faster biexciton decay, induced by the feedback, leads to considerably broader peaks in the spectrum. Thus, there exists a larger spectral overlap between the two light emission peaks centered at $\omega_X \pm \delta/2$ and it is therefore more probable that the two photons are emitted with the same frequency. In that case, the two decay paths are indistinguishable and the two photons are polarization entangled.

To give a systematic study, we show in Fig. 4(a) that an increased concurrence is achieved for a wide range of values for the fine-structure splitting $\delta$. This shows the robustness of our enhancement mechanism. The mechanism is also relatively robust against inhomogeneous broadening, e.g., by spectral diffusion. Simulations using a Gaussian distribution of $\hbar \omega_X$ with the FWHM = 20 $\mu$eV showed only a slight decrease of $\zeta$, since for the given values the feedback does not change strongly over the considered frequency range.

Going back to Fig. 2(b), and looking at the temporal evolution of the overlap $\zeta(t)$ for $\phi_X = \pi$, we see that first it decays rapidly after the feedback time $\tau_f$. This can be explained by a change of the peak width analogous to the above mechanism. However, after some time a temporal oscillation is visible. The frequency $\nu_{osc}$ of this oscillation is given by the fine-structure splitting via $\nu_{osc} = \delta/(2\pi) = \nu_\delta$.

FIG. 2 (color online). Dynamics of the biexciton cascade, including a fine structure splitting of $\delta = 10$ ns$^{-1} = 6.58 h^{-1}$ $\mu$eV. The three curves represent the cases without feedback (red, solid), with feedback phase $\phi_X = 0$ (green, dashed), and with feedback phase $\phi_X = \pi$ (blue, dotted). Subfigures show the results (a),(b) for a small feedback time ($\tau_f = 63$ ps) and (c),(d) for a larger feedback time ($\tau_f = 314$ ps). In (a) and (c) we plot the occupation probability of the biexciton state $|B\rangle$. In (a), after the feedback time $\tau_f$, the decay rates differ tremendously, depending on the feedback phase, while in (c), the dynamics are not affected by the feedback. In (b) and (d) we see the two-photon wave-function overlap $\zeta$, which converges to the concurrence $C$ for $t \to \infty$ [11,39,51]. For $\phi_X = 0$, $\zeta$ converges to a larger value than in the case without feedback. This effect is larger in the case of longer feedback delay. For $\phi_X = \pi$, $\zeta$ decreases rapidly after the feedback time. In the case of small $\tau_f$, pronounced oscillations in $\zeta$ are visible with a frequency given by the fine-structure splitting.

FIG. 3 (color online). Photon numbers $\langle \hat{c}^\dagger_k \hat{c}_k \rangle$ in the limit of $t \to \infty$. Gray shaded areas: without feedback; green line: (a) short (63 ps) and (b) longer (314 ps) delay times ($\phi_X = 0$). The two peaks originate from the fine-structure splitting. For short feedback delay, the peaks get broadened due to the faster decay rate [cf. Fig. 2(a)]. For longer feedback delay, the spectrum is more complex, with steeper flanks and a higher probability at $\omega = \omega_X$ than in the case without feedback.

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For large times, the amplitude of the oscillation decreases and $\zeta$ converges to a value slightly below 0.1 (not shown in the plot).

Results: (b) larger delay.—We will now examine feedback times $\tau_f$ comparable to the time scale given by the biexciton decay time $\tau_B$, which is also the time scale of the inverse fine-structure splitting $\nu_0^{-1}$. This scenario brings the feedback-induced modulation of the photon mode coupling in the same range as the fine-structure splitting. This circumstance creates a large difference in the feedback phase of the two emission peaks in the spectrum. In order to explore this regime, we increase the delay time to $\tau_f = \nu_0^{-1}/2 = 314$ ps, which is a factor of 5 larger than the delay time discussed in the last section. As we see in Fig. 2(c), the occupation probability of the biexciton level behaves very similarly with and without the mirror. This can be explained as follows: The feedback phase $\phi$ at the positions $\omega_X \pm \delta/2$ now differs by $\pi$. Hence, for $\omega_X = 0$ as well as for $\omega_X = \pi$, the phase $\phi(\omega_X \pm \delta)$ at the peaks of the spectrum is $\pi/2$ and $3\pi/2$, respectively. This explains the similarity in the biexciton decay dynamics, since the peaks are neither located in a node nor in an antinode, but right in the middle at the maximum slope.

In contrast, the normalized two-photon wave-function overlap $\zeta(t)$ differs strongly for the two different phases [cf. Fig. 2(d)]. For $\omega_X = \pi$, $\zeta(t)$ converges to a value similar to the value without feedback. For $\omega_X = 0$, we reach a concurrence $C$ of approximately 0.6, which is even higher than in the case of short feedback. We can explain this result by interference: photons with $\omega = \omega_X$ interfere constructively with their feedback field, while photons from the “flanks” of the spectrum interfere destructively. As depicted in Fig. 3(b), this steers the emission to the center of the spectrum, thereby increasing the probability of two photons of equal energy, which renders them polarization entangled. In Fig. 3(b), we clearly recognize the sharper edges of the spectrum and the increased photon probability at $\omega_X$ if feedback is applied. The mechanism at longer $\tau_f$ is apparently even more effective than just a broadening of the resonance peaks. As we can see in Fig. 4(b), the achieved concurrence for different delay times reaches a maximum at $\tau_f = 314$ ns, which is where the delay time matches the time scale given by $\delta$ via $\tau_f = \nu_0^{-1}/2 = \pi/\delta$. Therefore, we see that we can use $\omega_X$ as well as $\tau_f$ to control and optimize the amount of photon entanglement. However, the increased concurrence in the large delay regime comes with a trade-off: due to the strong frequency dependence of the feedback phase, the system is very sensitive to frequency fluctuations, e.g., by spectral diffusion, which could obliterate the effect. A possible solution might be resonant excitation [53]. Also, dephasing [54,55] degrades the possibility to observe the phenomenon and should therefore be reduced, e.g., by working in the low-temperature regime [56].

Conclusion.—We have demonstrated that the entanglement of photons emitted by a QD biexciton cascade with finite fine-structure splitting can be controlled and strongly enhanced by optical feedback. In the case of short feedback times, we could explain this by a stronger radiative decay rate. We showed that the concurrence can be increased even more by applying longer delay times $\tau_f$, an effect that we explain by a frequency-dependent interference of the photonic wave function with its feedback signal.

With the rise of millimeter-sized optics, the considered delay times are experimentally feasible. Shorter delay times can be reached with integrated optics [49]. Since we are dealing with an open system, and because the biexciton decay rate is not decreased for the feedback phase $\omega_X = 0$, we do not expect large detrimental effects on the entangled photon pair repetition rate. Successful proof-of-principle experiments of time-delayed feedback have been carried out in atom quantum optics [18,36]. We propose that it can be of great benefit to transfer this approach to a solid-state platform to create entangled photon pairs on demand.

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