

Cavity-assisted squeezing of a mechanical oscillator

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We investigate the creation of squeezed states of a vibrating membrane or a movable mirror in an opto-mechanical system. An optical cavity is driven by squeezed light and couples via radiation pressure to the membrane/mirror, effectively providing a squeezed heat-bath for the mechanical oscillator. Under the conditions of laser cooling to the ground state, we find an efficient transfer of squeezing with roughly 60% of light squeezing conveyed to the membrane/mirror (on a dB scale). We determine the requirements on the carrier frequency and the bandwidth of squeezed light. Beyond the conditions of ground state cooling, we predict mechanical squashing to be observable in current systems.

I. INTRODUCTION

Recent progress in feedback and cavity-assisted cooling of micro- and nano-mechanical resonators [1, 2, 3, 4, 5, 6, 7, 8, 9] shows that opto-mechanical systems are ultimately approaching the quantum regime. Occupancy levels of around 30 quanta of a motional mode of a vibrating mirror have already been achieved experimentally [9] via the sideband cooling technique and limitations at the moment seem to be of a rather technical nature. Similar to the case of a light field where the imprint of the quantum regime is signaled by the generation of nonclassical states such as squeezed states, generation of squeezing of a nano-mechanical object can be a hallmark for quantum control of a macroscopic, massive object [10, 11, 12]. On a more practical side, nano-mechanical squeezing might have applications in ultrahigh precision measurement experiments [13] and detection of gravitational waves [14].

Experimental squeezing of a nano-mechanical object was so far only achieved for a nonlinear Duffing resonator [15]. On the theory side, squeezing of a linear nano-mechanical resonator was proposed via coupling to an auxiliary nonlinear system, such as an optical cavity containing an atomic medium [16], a SQUID loop [17, 18] or a Cooper-pair box circuit [19]. In principle squeezing could also be generated by teleportation of squeezed states of light [20] or atoms [21] to the mechanical resonator. More direct approaches in opto-mechanical settings are based on modulated or parametric drive with or without feedback [22, 23, 24, 25], or similarly, modulated readout combined with a feedback loop [26].

In this paper we analyze a scheme for generation of squeezing via reservoir engineering in an opto-mechanical setup. Namely, starting from a laser-cooled membrane, by accompanying the cooling beam with a much weaker, squeezed vacuum input field, we show that the membrane motion is driven into the steady-state of a squeezed environment, which is a squeezed state. This method does not require feedback or modulation of drive fields. It combines naturally with requirements for ground state

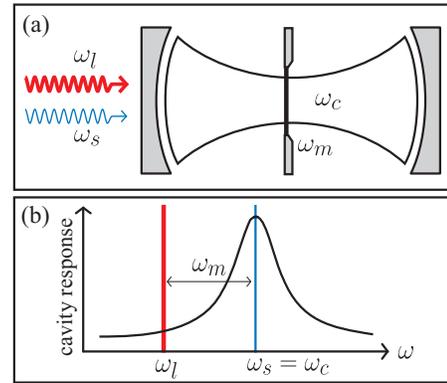


FIG. 1: (a) A mechanical mode of a dielectric membrane, oscillating at frequency ω_m , is coupled via radiation pressure to the cavity field of frequency ω_c . The cavity is driven by a laser of frequency ω_l and, in addition, a much weaker squeezed vacuum field with central frequency ω_s . (b) Optimal choice for the frequencies ω_l , ω_s , in order to transfer squeezing from the light field to the membrane motion: the laser frequency is chosen such that the cooling sideband is addressed, i.e. $\omega_l = \omega_c - \omega_m$, whereas the center frequency of the squeezing has to equal the cavity resonance frequency ω_c .

cooling, avoiding drive fields on the blue side of the cavity resonance and associated issues concerning dynamical stability. We show that, under the conditions of ground state cooling, a significant transfer of squeezing from light to mechanical degrees of freedom is possible. As a rule of thumb, our studies predict a transfer of approximately 60% of squeezing on a logarithmic scale, e.g. 6 dB of light squeezing would result in 4 dB of mechanical squeezing. Optimal transfer is achieved when the central frequency of squeezing is resonant with the cavity mode. This resonance condition has to be fulfilled within a tolerance on the order of the spectrally broadened linewidth of the mechanical oscillator, which is typically smaller than the cavity linewidth. We take full account of a finite

bandwidth of squeezed light, which makes the dynamics essentially non-Markovian. We find that an optimal finite bandwidth exists if the sidebands are only poorly resolved, while there is practically no dependence on bandwidth in the resolved sideband limit. Finally, we show that beyond the regime of ground state cooling squashed states [27] of the mechanical resonator could be observed under present experimental conditions.

The transfer of quantum noise properties from the radiation field to the mechanical degrees of freedom lies also at the heart of the schemes presented in [28, 29], which aim for the creation of entanglement between two movable mirrors. With experiments entering the quantum regime of cooling close to the ground state, we see the present work as a natural first step towards such more demanding protocols. We provide here for the first time a careful analysis of the effects of mismatch in resonance conditions and finite squeezing bandwidth.

The paper is structured as follows: In Section II we derive the exact equations of motion (quantum Langevin equations) of the coupled light-membrane system. In Section III we derive the optimal conditions for squeezing transfer and find simple analytical expressions for the membrane variances in an adiabatic limit where the cavity field is eliminated and under the assumption of resolved-sideband limit. We compare the results with an exact numerical solution and analyze the domain of validity of the adiabatic elimination of the field. Conclusions are presented in Section IV and analytical results outside the resolved sideband limit are listed in the Appendix.

II. OPTO-MECHANICAL SYSTEM DRIVEN BY SQUEEZED LIGHT

We consider the dispersive opto-mechanical system illustrated in Fig. 1, where a vibrating dielectric membrane is placed between the two fixed end-mirrors of a laser-driven, single sided Fabry-Perot cavity [6]. The theory presented is in principle also applicable to a Fabry-Perot cavity with a light, movable end-mirror [30], or to a vibrational mode of a whispering gallery mode cavity [31, 32]. The opto-mechanical coupling strength in general depends on the particular geometrical factors of the system under consideration while a prototypical system can always be modeled by a simple Hamiltonian

$$H = \hbar\omega_c c^\dagger c + \hbar\omega_{m0} b^\dagger b - \hbar g c^\dagger c (b^\dagger + b) + i\hbar (E e^{-i\omega_1 t} c^\dagger - E^* e^{i\omega_1 t} c). \quad (1)$$

The first term in the Hamiltonian gives the energy of a single cavity mode at an optical frequency ω_c , characterized by the annihilation operator c satisfying the commutation relation $[c, c^\dagger] = 1$. The second term gives the energy of the motional mode of the membrane, at a resonance frequency ω_{m0} , where b is the annihilation operator for vibrational quanta. The third term describes the membrane-light radiation pressure coupling

with strength $g = 2R(\bar{x}_m/L)\omega_c$, where $\bar{x}_m = \sqrt{\hbar/2m\omega_{m0}}$ is the zero-point motion of a membrane mode of effective mass m . The geometrical factors L and R are the effective cavity length and reflectivity of the membrane. In our setup the membrane is placed at the point of maximal linear coupling to the cavity field [6]. The last term shows driving of the cavity field with a laser at frequency ω_1 and strength E related to the laser power P by $|E| = \sqrt{2P\kappa/\hbar\omega_c}$, where κ is the cavity amplitude decay rate.

In addition to the evolution described by the Hamiltonian in Eq. (1) the system is also subjected to random noise forces due to fluctuations of the phononic heat bath of the membrane and to quantum fluctuations of the radiation field. The main idea here is to deliberately shape the noise properties of the latter bath by driving the cavity with a squeezed vacuum field in parallel to the coherent field. The effect of squeezed vacuum noise on the cavity field is included in a non-Markovian noise model with damping rate κ and input noise operator $c_{in}(t)$ whose two-time correlation functions are

$$\begin{aligned} \langle \bar{c}_{in}(t + \tau) \bar{c}_{in}(t) \rangle &= \frac{M}{2} \frac{b_x b_y}{b_x^2 + b_y^2} \left(b_y e^{-b_x |\tau|} + b_x e^{-b_y |\tau|} \right) \\ \langle \bar{c}_{in}^\dagger(t + \tau) \bar{c}_{in}(t) \rangle &= \frac{N}{2} \frac{b_x b_y}{b_x^2 - b_y^2} \left(b_y e^{-b_x |\tau|} - b_x e^{-b_y |\tau|} \right). \end{aligned} \quad (2)$$

The noise operators $\bar{c}_{in}(t) = e^{i\omega_s t} c_{in}(t)$ refer to a frame rotating at the carrier frequency of squeezing ω_s and satisfy the canonical commutation relation $[\bar{c}_{in}(t), \bar{c}_{in}^\dagger(t')] = \delta(t - t')$ [33]. Parameters N and M determine the degree of squeezing, while b_x and b_y define the squeezing bandwidth. The connection of these parameters to the properties of the optical parametric oscillator cavity generating the squeezing are summarized in Appendix A. In particular for pure squeezing there are only two independent parameters, as in this case $|M|^2 = N(N + 1)$ and $b_y = b_x \sqrt{2(N + |M|) + 1}$. While we will derive all results for a finite bandwidth, it will be instructive and more transparent to consider in certain cases the white noise limit for squeezing, that is $b_x, b_y \rightarrow \infty$ while keeping N, M constant. In this case the correlation functions of Eq. (2) are simplified to $\langle \bar{c}_{in}(t + \tau) \bar{c}_{in}(t) \rangle \rightarrow M\delta(\tau)$ and $\langle \bar{c}_{in}^\dagger(t + \tau) \bar{c}_{in}(t) \rangle \rightarrow N\delta(\tau)$.

The thermal bath affecting the motion of the membrane is Brownian and consequently non-Markovian [34]. However, in the high temperature limit ($2kT \gg \hbar\omega_{m0}$), which is applicable to this system even for cryogenic temperatures, the noise can be described using a Markovian model with membrane loss rate γ_m . The noise operators $b_{in}(t)$ are characterized by the correlation functions

$$\langle b_{in}^\dagger(t) b_{in}(t') \rangle = n_{th} \delta(t - t') \quad (3)$$

and commutation relations $[b_{in}(t), b_{in}^\dagger(t')] = \delta(t - t')$. Here we denoted by n_{th} the average thermal occupation number of the mechanical mode.

Having specified the Hamiltonian and noises affecting the membrane-light system, we proceed now to analyze its dynamics using the quantum Langevin equations formalism

$$\begin{aligned}\dot{c} &= -(i[\omega_c - g(b^\dagger + b)] + \kappa)c + Ee^{-i\omega_1 t} + \sqrt{2\kappa}c_{\text{in}}(t) \\ \dot{b} &= -(i\omega_{\text{m}0} + \gamma_{\text{m}})b + igc^\dagger c + \sqrt{2\gamma_{\text{m}}}b_{\text{in}}(t).\end{aligned}\quad (4)$$

In the following we perform a transformation to a frame rotating at the laser frequency ω_1 , so that $\tilde{c}(t) = c(t)e^{i\omega_1 t}$. We linearize operators around the steady state values $\tilde{c} = \langle \tilde{c} \rangle_{\text{ss}} + \delta\tilde{c}$, $b = \langle b \rangle_{\text{ss}} + \delta b$ such that the fluctuations δb and $\delta\tilde{c}$ have zero mean. We find the steady state value for the cavity amplitude $\langle \tilde{c} \rangle_{\text{ss}} = E/(\kappa + i\Delta)$, where the effective detuning is

$$\Delta = \omega_c - \omega_1 - \frac{2g^2\omega_{\text{m}0}}{\omega_{\text{m}0}^2 + \gamma_{\text{m}}^2} \langle \tilde{c}^\dagger \tilde{c} \rangle_{\text{ss}}.$$

For simplicity we take $\langle \tilde{c} \rangle_{\text{ss}}$ to be real and positive; this can be achieved by an appropriate choice of the laser phase. Similarly, we find for the mechanical oscillator $\langle b \rangle_{\text{ss}} = (g/(\omega_{\text{m}0} - i\gamma_{\text{m}})) \langle \tilde{c}^\dagger \tilde{c} \rangle_{\text{ss}}$. Let us remark, that the intra-cavity occupation at the steady state, $\langle \tilde{c}^\dagger \tilde{c} \rangle_{\text{ss}}$, contains two contributions, one from the laser drive and the other from inserting squeezed light. The laser contributes a number of $|\langle \tilde{c} \rangle_{\text{ss}}|^2$ photons, which – in present setups – are typically more than 10^5 photons. One can show, that the contribution from squeezed light is given by $N(b_x b_y / (b_y^2 - b_x^2)) [b_y / (b_x + \kappa) - b_x / (b_y + \kappa)] \leq N$. Squeezed vacuum exhibiting a large noise reduction of -10 dB below shot noise level [35] thus contributes at most $N \sim 2$ intra-cavity photons. The contribution of the squeezed vacuum to the intra-cavity photon number is thus negligible.

We can proceed now to linearize the equations of motion (4) following the standard treatments of opto-mechanical coupling [30]. A mean field expansion around the large coherent intracavity amplitude $\langle \tilde{c} \rangle_{\text{ss}}$ yields a linear set of equations

$$\begin{aligned}\delta\dot{\tilde{c}} &= -(i\Delta + \kappa)\delta\tilde{c} + \frac{iG}{2}(\delta b^\dagger + \delta b) + \sqrt{2\kappa}c_{\text{in}}(t)e^{i\Delta_s t} \\ \delta\dot{b} &= -(i\omega_{\text{m}0} + \gamma_{\text{m}})\delta b + \frac{iG}{2}(\delta\tilde{c}^\dagger + \delta\tilde{c}) + \sqrt{2\gamma_{\text{m}}}b_{\text{in}}(t)\end{aligned}\quad (5)$$

where we introduced the detuning between the laser and the center frequency of the squeezing

$$\Delta_s = \omega_1 - \omega_s$$

and the effective opto-mechanical coupling

$$G = 2g\langle \tilde{c} \rangle_{\text{ss}}.$$

Finally we move to a frame rotating at the detuning Δ for cavity operators and rotating at a *shifted* mechanical frequency $\omega_{\text{m}} = \omega_{\text{m}0} - \Omega$ for the membrane, i.e. we

transform $\delta c_{\text{I}}(t) = \delta\tilde{c}(t)e^{i\Delta t}$ and $\delta b_{\text{I}}(t) = \delta b(t)e^{i\omega_{\text{m}} t}$. We use the shifted frequency ω_{m} to accommodate for the optical spring effect [30], which introduces a shift Ω of the bare resonance frequency $\omega_{\text{m}0}$, as we will see later on. Typically this shift is negligible for oscillators at the level of MHz, but it turns out to play an important role in the condition of optimal squeezing transfer. The exact expression of Ω will be detailed in the next Section. The quantum Langevin equations that form the starting point of our analysis can now be written

$$\begin{aligned}\delta\dot{c}_{\text{I}} &= -\kappa\delta c_{\text{I}} + \frac{iG}{2}e^{i\Delta t}(\delta b_{\text{I}}^\dagger e^{i\omega_{\text{m}} t} + \delta b_{\text{I}} e^{-i\omega_{\text{m}} t}) \\ &\quad + \sqrt{2\kappa}c_{\text{in}}(t)e^{i(\Delta + \Delta_s)t}\end{aligned}\quad (6)$$

$$\begin{aligned}\delta\dot{b}_{\text{I}} &= -(\gamma_{\text{m}} + i\Omega)\delta b_{\text{I}} + \frac{iG}{2}e^{i\omega_{\text{m}} t}(\delta c_{\text{I}}^\dagger e^{i\Delta t} + \delta c_{\text{I}} e^{-i\Delta t}) \\ &\quad + \sqrt{2\gamma_{\text{m}}}b_{\text{in,I}}(t).\end{aligned}\quad (7)$$

III. MECHANICAL SQUEEZING

The set of Eqs. (6, 7) can, in principle, be solved exactly. However, the resulting expressions are rather cumbersome and do not offer the necessary physical insight into the transfer of squeezing from the light to the motion of the membrane. Consequently we derive an approximate solution in the perturbative limit of weak opto-mechanical coupling $G \ll \kappa$, which we then compare with the exact solution of Eqs. (6,7) for particular sets of parameters.

As a first main step in our analysis, we note that squeezing one of the membrane's variances below the shot noise limit can only be expected under conditions where the state of the membrane is already close to the ground state. Therefore, we require simultaneous laser cooling of the motion which can be achieved with the condition

$$\Delta = \omega_{\text{m}},$$

i.e., in the sideband cooling regime [30, 36, 37].

Under the assumption of fast cavity dynamics on the time scale of the cavity-membrane coupling, $G/\kappa \ll 1$, we adiabatically eliminate the cavity mode (see also [36]). Formally solving for the cavity dynamics (6), and keeping terms up to $O(G/\kappa)$ when evaluating the integrals over the membrane motion, we find an approximate solution for the cavity mode $\delta c_{\text{I}}(t)$,

$$\begin{aligned}\delta c_{\text{I}}(t) &= \frac{iG}{2\kappa}\delta b_{\text{I}}(t) + \frac{iG}{2(\kappa + 2i\omega_{\text{m}})}e^{2i\omega_{\text{m}} t}\delta b_{\text{I}}^\dagger(t) \\ &\quad + \sqrt{2\kappa}\int_0^t d\tau e^{-\kappa(t-\tau)}e^{i(\omega_{\text{m}} + \Delta_s)\tau}c_{\text{in}}(\tau).\end{aligned}\quad (8)$$

Inserting the solution Eq. (8) into the membrane equation of motion, Eq. (7), the result is an effective equation

of motion for the membrane

$$\begin{aligned} \delta \dot{b}_I(t) = & -\gamma_{\text{eff}} \delta b_I(t) + (\Gamma + i\Omega) e^{2i\omega_m t} \delta b_I^\dagger(t) + \sqrt{2\gamma_m} b_{\text{in},I}(t) \\ & + \frac{iG\sqrt{\kappa}}{\sqrt{2}} \int_0^t d\tau e^{-\kappa(t-\tau)} \left[e^{i(\omega_m + \Delta_s)\tau} \bar{c}_{\text{in}}(\tau) \right. \\ & \left. + e^{2i\omega_m t} e^{-i(\omega_m + \Delta_s)\tau} \bar{c}_{\text{in}}^\dagger(\tau) \right]. \quad (9) \end{aligned}$$

The first term on the right hand side describes damping at an effective decay rate,

$$\gamma_{\text{eff}} = \gamma_m + \Gamma,$$

which is the sum of the intrinsic decay rate γ_m of the membrane due to its coupling to the thermal bath, and the radiation pressure induced decay rate Γ , defined by

$$\Gamma = \frac{G^2}{4\kappa} \frac{4\omega_m^2}{\kappa^2 + 4\omega_m^2}.$$

The enhanced decay rate γ_{eff} is the signature of sideband cooling. For high Q mechanical oscillators and efficient laser cooling we have $\Gamma \gg \gamma_m$ such that we will take $\gamma_{\text{eff}} \simeq \Gamma$ in the following. The adiabatic elimination of the cavity mode also gives rise to a shift Ω of the membrane frequency, which is given by $\Omega = \Gamma(2\kappa/\omega_m)$. The second term on the right hand side of Eq. (9) is a counter-rotating term; it makes a negligible contribution as $\Omega, \Gamma \ll \omega_m$ under the present assumptions, consequently we can neglect it in the following.

The last term in Eq. (9) corresponds to the desired squeezed, non-Markovian Langevin force driving the motion of the membrane. We are interested in an optimal transfer of the squeezing properties to the mechanical system. An important question is, how the center frequency of the squeezing ω_s is to be chosen with respect to the cavity frequency. In order to get an efficient coupling of b_I to the squeezed noise force \bar{c}_{in} , the time dependence of the integrand in the last term of Eq. (9) suggests the choice

$$\Delta_s = -\omega_m,$$

which amounts to a squeezing spectrum centered at the cavity resonance frequency $\omega_s = \omega_c$. The coupling to $\bar{c}_{\text{in}}^\dagger$ will then be fast oscillating and make a negligible contribution. Below we will give a more physical picture for this resonance condition along with a discussion of the tolerance to a mismatch in this condition.

In the following we restrict our discussion to the resolved sideband limit case, i.e. when the resolved sideband parameter is $\eta = \kappa/\omega_m \ll 1$. In this regime, cooling of the membrane is optimal and efficient squeezing is expected. For simplicity of presentation, we keep only the zeroth order terms in η , which results in a simple form for the expression of the optical cooling rate $\Gamma = G^2/(4\kappa)$. The full analytical expressions valid beyond the resolved

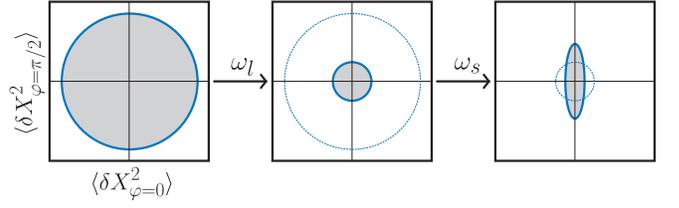


FIG. 2: Schematic phase-space picture of mechanical squeezing via reservoir engineering with squeezed light: A mechanical resonator in thermal equilibrium with equal variance in conjugate quadratures $\delta X_{\varphi=0}$ and $\delta X_{\varphi=\pi/2}$ (left) is laser sideband cooled by a coherent driving field at frequency ω_l reduces the mechanical variances $\langle \delta X_{\varphi} \delta X_{\varphi} \rangle$ close to the ground state variances (center). A second, squeezed vacuum field at frequency ω_s drives the system into a state with (anti)squeezed variances in conjugate variables (right).

sideband limit are presented in Appendix B. The solution of Eq. (9) in the long time limit is

$$\delta b_I(t) = i\delta b_{\text{I,sq}}(t) + \sqrt{\frac{\gamma_m}{\gamma_{\text{eff}}}} \delta b_{\text{I,th}}(t) \quad (10)$$

and exhibits a squeezing and a thermal contribution. The contribution due to the radiation pressure coupling to the squeezed environment is

$$\begin{aligned} \delta b_{\text{I,sq}}(t) = & \sqrt{2\gamma_{\text{eff}}\kappa} \int_0^t d\tau e^{-\gamma_{\text{eff}}(t-\tau)} \int_0^{\tau} d\tau' e^{-\kappa(\tau-\tau')} \\ & \times \left[e^{2i\omega_m\tau} \bar{c}_{\text{in}}^\dagger(\tau') + \bar{c}_{\text{in}}(\tau') \right] \quad (11) \end{aligned}$$

and the thermal contribution is

$$\delta b_{\text{I,th}}(t) = \sqrt{2\gamma_{\text{eff}}} \int_0^t d\tau e^{-\gamma_{\text{eff}}(t-\tau)} b_{\text{in},I}(\tau). \quad (12)$$

The two noise processes are uncorrelated such that their effects to the steady state statistics will simply add up.

In order to see the effect of squeezing we need to evaluate the variances of the generalized quadrature operator

$$\delta X_{\varphi,I}(t) = \frac{1}{\sqrt{2}} \left(e^{i\varphi} \delta b_I(t) + e^{-i\varphi} \delta b_I^\dagger(t) \right), \quad (13)$$

which specializes for $\varphi = 0$ to the usual position operator $\delta q_I(t)$ and for $\varphi = -\pi/2$ to the momentum operator $\delta p_I(t)$, both taken in a rotating frame at frequency ω_m . In Appendix B we evaluate the quadrature correlations for finite bandwidth squeezing, that is when $\delta b_{\text{I,sq}}(t)$ is a non-Markovian noise process. The result is very intuitive in the limit of squeezed white noise where the quadrature correlations take a simple form

$$\begin{aligned} \langle \delta X_{\varphi,I}(t) \delta X_{\varphi,I}(t) \rangle = & \left(N + \frac{1}{2} - \text{Re} \{ M e^{2i\varphi} \} \right) \\ & + \frac{\gamma_m}{\gamma_{\text{eff}}} \left(n_{\text{th}} + \frac{1}{2} \right). \quad (14) \end{aligned}$$

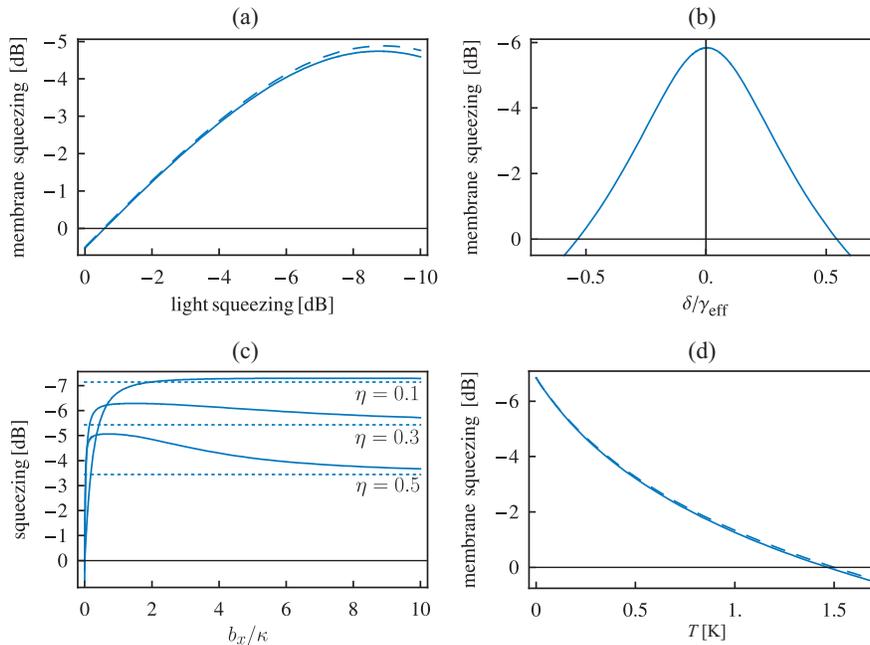


FIG. 3: (a) Efficiency of squeezing transfer from light to the membrane (in dB) for the following set of parameters $Q = 10^7$, $m = 1$ ng, $\omega_{m0} = 2\pi \times 1$ MHz, $\kappa = 2\pi \times 380$ kHz, $G = 2\pi \times 110$ kHz and $T = 100$ mK. The analytical result of Eq. (14) (dashed line) agrees well with the exact numerical result (full line). (b) Squeezing of the membrane vs. deviations from the resonance, $\delta = \Delta_s + \omega_m$. Mechanical squeezing requires the resonance condition $\delta = 0$ to be fulfilled within the effective mechanical linewidth γ_{eff} . (c) Squeezing transfer vs. bandwidth of squeezing b_x for various values of the sideband parameter $\eta = \kappa/\omega_m$ (full lines). Dashed lines give the corresponding value for white squeezed input noise. Outside the resolved sideband regime the bandwidth plays a role in optimizing the transfer of squeezing. (d) Dependence of the generated motional squeezing on the membrane environment temperature.

The interpretation of Eq. (14) is quite straightforward. In the first term we recognize the squeezing properties of the input field

$$\langle x_{\varphi, \text{in}}(t)x_{\varphi, \text{in}}(t - \tau) \rangle = \left(N + \frac{1}{2} + \text{Re}\{M e^{2i\varphi}\} \right) \delta(\tau),$$

where the light quadratures are $x_{\varphi, \text{in}}(t) = \frac{1}{\sqrt{2}} \left(e^{i\varphi} \bar{c}_{\text{in}}(t) + e^{-i\varphi} c_{\text{in}}^\dagger(t) \right)$ and we used Eq. (2) in the white noise limit. The second term is the thermal variance, suppressed by a factor γ_m/γ_{eff} , as is familiar from the opto-mechanical laser sideband cooling [30, 36, 37]. In fact, for the particular case of no squeezing ($M = N = 0$), the final occupation number $n_f = (1/2)(\langle \delta q_I(t)^2 \rangle + \langle \delta p_I(t)^2 \rangle - 1)$ is given by $n_f \simeq \kappa^2 / (2\omega_{m0})^2 \ll 1$, which is the well-known residual occupancy for resolved sideband cooling as obtained in Refs. [30, 36, 37]. Figure 2 provides a phase space illustration of our result.

The simple result Eq. (14) is valid only in the limit of squeezed white noise and to zeroth order in the sideband parameter η . For a realistic discussion of the main obstacles to obtain perfect squeezing, we extend our treatment beyond the resolved-sideband regime. Our first observation is that for a nonzero η , even for a large cooling efficiency (where we can still neglect the contribution of

the phononic heat bath), the resulting squeezed state is thermal. We find that N of the pure squeezed state in (14) is replaced by $N' = N[1 + \eta/(\eta + 4)] + \eta/(2[\eta + 4])$ while M is unchanged. For the resulting squeezed mechanical state we thus have $|M|^2 \leq N'(N' + 1)$, which is the signature of a mixed, squeezed state. Thus, for a given finite sideband parameter η , we expect that for very large squeezing of light, mechanical squeezing eventually starts to degrade due to an increasing impurity.

In Fig. 3 we compare the analytical results for arbitrary bandwidth squeezing, given by Eq. (B1) and squeezed white noise, given by Eq. (B2) with numerical results obtained by exactly solving the equation of motion Eqs. (6, 7) in steady state. In Fig. 3a, we study the squeezing transferred to the membrane as a function of the squeezing of the input light (in dB). The results obtained via the exact numerical method are compared with the approximate results of Eq. (14). Under the conditions of ground state cooling, we see an efficient transfer of squeezing, with about 60% of light squeezing being transferred to the mechanical system. As expected for the reasons given above, for a high amount of light squeezing the transfer degrades again.

Of great importance is the sensitivity of the transfer to deviations from the resonance conditions. It is well known [38] that cavity cooling is fairly robust to devia-

tions from the optimal cooling condition $\Delta = \omega_m$. Here we find, as shown in Fig. 3b where the membrane squeezing is plotted vs. the detuning $\delta = \Delta_s + \omega_m$, that fairly small deviations (of order γ_{eff}) from the condition of resonant squeezing transfer lead to washed out membrane squeezing. This result can be understood from the fact that γ_{eff} gives the bandwidth of the mechanically scattered cooling sideband, such that a deviation $\delta > \gamma_{\text{eff}}$ would mean that the central frequency of the squeezing input completely misses the sideband cooling spectrum.

The results shown in Fig. 3c indicate that in general there is an optimal squeezing bandwidth for which the transfer from light to membrane is maximized, but for small η the finite bandwidth result of Eq. (B1) (full lines) does not differ much from the infinite bandwidth limit result (dashed lines). For a large bandwidth which fully covers the motional sidebands, $b_x \gg \omega_m$, the membrane sees only white squeezed input noise, whereas for smaller bandwidth, the crucial question is whether the squeezed input will touch the heating sideband or not. For small η , as we see in Fig. 3c, the width is not a big issue, since the heating sideband is anyway weak, whereas for large η the squeezing transfer is much improved for an optimal, finite bandwidth where the strong heating sideband is avoided. For the same reason, the ratio of the optimal bandwidth to κ decreases for large η .

Finally, we investigated the squeezing transfer as a function of the environmental temperature. As shown in Fig. 3d it is of great importance to provide a cold environment where ground-state cooling of the membrane is allowed. However, as ground-state cooling of membranes has yet to be demonstrated, we investigated the squashing [27] of the membrane state for existing experimental parameters. In the context of existing cavity-assisted mirror/membrane cooling experiments an occupancy of around $n_f = 10$ is in sight. This can be achieved for the set of parameters $Q = 10^7$, $m = 10$ ng, $\omega_{m0} = 2\pi \times 1$ MHz, $\kappa = 2\pi \times 125$ kHz, $G = 2\pi \times 21$ kHz and $T = 4$ K, and corresponding cavity finesse of 6×10^4 , cavity length $L = 20$ mm and a circulating power of $P = 3$ mW. For a squashed membrane state transferred from 6 dB of squeezed vacuum input light, the imbalance in variances is about 20%. This imbalance will be sensitive to the phase of the squeezed vacuum input and should be easily detectable for the present shot-noise limited detection efficiencies [39]. The basic principle of state engineering of a mechanical oscillator via reservoir engineering of quantum fluctuations of the radiation field, should thus be readily observable.

IV. CONCLUSION

In conclusion, we have analyzed the conditions under which transfer of squeezing from light to a membrane within an optical cavity is possible and optimal. We have found that the best operating regime is in the typical resolved-sideband limit for opto-mechanical cooling,

and that optimal transfer occurs when the squeezed light is fed into the cavity at resonance. We have also shown that the squeezed light bandwidth is a factor in the efficiency of the transfer when one operates outside the strictly resolved-sideband limit. However, we conclude that under ideal cooling conditions, as long as the squeezing bandwidth exceeds the laser-induced cooling rate of the membrane, the squeezed white noise description is perfectly valid.

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APPENDIX A: PARAMETRIZATION OF SQUEEZED LIGHT

A squeezed light field described by the correlation functions (2) is routinely produced in an optical parametric oscillator (OPO) driven below threshold [40]. The parameters M, N and bandwidths b_x, b_y of the squeezed light field (2) are related to the susceptibility ϵ of the OPO and the damping rate γ of the OPO cavity by $b_x = \gamma/2 - |\epsilon|$ and $b_y = \gamma/2 + |\epsilon|$, and by $M = (\epsilon\gamma/2) [1/b_x^2 + 1/b_y^2]$ and $N = (|\epsilon|\gamma/2) [1/b_x^2 - 1/b_y^2]$. From this definition it is clear that $N \geq 0$; furthermore, the stability of the OPO cavity requires $b_x \geq 0$. The chosen parametrization satisfies the relation for a pure squeezed state, $|M|^2 = N(N+1)$, which is the maximum squeezing limit of the general property $|M|^2 \leq N(N+1)$ for squeezed noise. Note that two of the parameters are redundant for the given case, as discussed in Section II.

APPENDIX B: BEYOND THE RESOLVED SIDEBAND LIMIT

Here we present the analytical results of Section III generalized to arbitrary ratios of η . The adiabatic elimination of the cavity mode leaves us with the effective equation of motion (9) for the membrane mode and its solution (10) which here generalizes to,

$$\delta b_{\text{I}}(t) = \frac{iG}{\sqrt{\gamma_{\text{eff}}\kappa}} \frac{\omega_m}{\sqrt{\kappa^2 + 4\omega_m^2}} \delta b_{\text{I,sq}}(t) + \sqrt{\frac{\gamma_m}{\gamma_{\text{eff}}}} \delta b_{\text{I,th}}(t).$$

The contribution due to the squeezed environment is now given by (cf. Eq. (11)),

$$\delta b_{\text{I,sq}}(t) = \sqrt{2\gamma_{\text{eff}}\kappa} \sqrt{1 + \frac{\kappa^2}{4\omega_m^2}} \int_0^t d\tau e^{-\gamma_{\text{eff}}(t-\tau)} \int_0^\tau d\tau' e^{-\kappa(\tau-\tau')} \left[e^{2i\omega_m\tau} \bar{c}_{\text{in}}^\dagger(\tau') + \bar{c}_{\text{in}}(\tau') \right],$$

whereas the thermal contribution (12) remains unchanged. Evaluating the correlations of the generalized quadrature operators (13) gives the result,

$$\begin{aligned} \langle \delta X_{\varphi, I}(t), \delta X_{\varphi, I}(t) \rangle = & \\ & \frac{G^2}{4\gamma_{\text{eff}}\kappa} \left[Nf_- + \frac{1}{2} + \frac{\kappa^2}{\kappa^2 + 4\omega_m^2} \left\{ Nh + \frac{1}{2} \right\} \right. \\ & \left. - \text{Re}\{Me^{2i\varphi}\}f_+ \right] + \frac{\gamma_m}{\gamma_{\text{eff}}} \left(n_{\text{th}} + \frac{1}{2} \right), \quad (\text{B1}) \end{aligned}$$

where the first two lines describe the contribution from the squeezed environment, whereas the last line contains the thermal contribution. For clarity, the cumbersome bandwidth dependence is contained in the coefficients f_{\pm} and h ,

$$\begin{aligned} f_{\pm} &= \frac{b_x b_y}{b_y^2 \pm b_x^2} \left[\frac{b_y}{b_x + \gamma_{\text{eff}}} \pm \frac{b_x}{b_y + \gamma_{\text{eff}}} \right], \\ h &= \frac{b_x b_y}{b_y^2 - b_x^2} \left[\frac{b_y [b_x^2 + b_x \kappa + \frac{\gamma_{\text{eff}}}{\kappa}(\kappa^2 + 4\omega_m^2)]}{(b_x + \kappa)(b_x^2 + 2b_x \gamma_{\text{eff}} + 4\omega_m^2)} \right. \\ & \quad \left. - \frac{b_x [b_y^2 + b_y \kappa + \frac{\gamma_{\text{eff}}}{\kappa}(\kappa^2 + 4\omega_m^2)]}{(b_y + \kappa)(b_y^2 + 2b_y \gamma_{\text{eff}} + 4\omega_m^2)} \right]. \end{aligned}$$

In the white noise limit these functions simplify to $h, f_{\pm} \rightarrow 1$ and we obtain

$$\begin{aligned} \langle \delta X_{\varphi, I}(t), \delta X_{\varphi, I}(t) \rangle = & \\ & \frac{G^2}{4\gamma_{\text{eff}}\kappa} \left[\left(N + \frac{1}{2} \right) \left(1 + \frac{\kappa^2}{\kappa^2 + 4\omega_m^2} \right) - \text{Re}\{Me^{2i\varphi}\} \right] \\ & + \frac{\gamma_m}{\gamma_{\text{eff}}} \left(n_{\text{th}} + \frac{1}{2} \right). \quad (\text{B2}) \end{aligned}$$

Deep in the resolved sideband limit, keeping only terms up to zeroth order in κ/ω_m , we recover the simple result (14).

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