

Alkaline-Earth-Metal Atoms as Few-Qubit Quantum Registers

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We propose and analyze a novel approach to quantum information processing, in which multiple qubits can be encoded and manipulated using electronic and nuclear degrees of freedom associated with individual alkaline-earth-metal atoms trapped in an optical lattice. Specifically, we describe how the qubits within each register can be individually manipulated and measured with subwavelength optical resolution. We also show how such few-qubit registers can be coupled to each other in optical superlattices via conditional tunneling to form a scalable quantum network. Finally, potential applications to quantum computation and precision measurements are discussed.

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Stringent requirements on the implementation of scalable quantum information systems can be significantly relaxed if the entire system can be subdivided into smaller quantum registers, in which several qubits can be stored for a long time and local quantum operations can be carried out with a very high fidelity [1–3]. In such a case, scalable quantum networks can be built even if the nonlocal coupling between registers has either low fidelity or is probabilistic [1–3]. Local operations can be achieved with the highest fidelity if the entire register is encoded into a single atom or molecule. While quantum registers based on individual solid-state impurities are already being explored [4], typical qubits encoded into hyperfine [5] or Rydberg [6] states of isolated atoms cannot be easily used as a few-qubit register. More specifically, the Hilbert space associated with such systems cannot be represented as a direct product of several subsystems, such that, e.g., one of the qubits can be measured without affecting the others.

In this Letter, we show that individual alkaline-earth-metal atoms can be used for the robust implementation of quantum registers, with one (electronic) qubit encoded in a long-lived optical transition and several additional qubits encoded in the nuclear spin. Following the proposal of Ref. [2], we use the electronic qubit as the communication qubit [3,4] for detecting and coupling the registers. In particular, we show that the full $(2I + 1)$ -dimensional space describing a spin- I nucleus can be preserved during electronic-qubit detection. This step uses off-resonant detection proposed in Ref. [2] and extends the proposal of Ref. [7] beyond one nuclear qubit manipulation to much larger registers (I can be as high as $9/2$, as in ^{87}Sr). We also show how to manipulate and measure individual registers in an optical lattice with subwavelength resolution. While entangling gates between alkaline-earth-metal atoms have been studied in the context of nuclear qubits alone [8,9] and electronic qubits alone [10], we propose a new scheme that makes use of both degrees of freedom. Our gate creates

entangled states between electronic qubits using conditional resonant tunneling and an interaction blockade [11,12].

The register.—Figure 1(a) shows, as an example, the relevant alkaline-earth-like structure of ^{171}Yb ($I = 1/2$). We want to arbitrarily manipulate the $2(2I + 1)$ -dimensional Hilbert space consisting of states in the manifolds $|g\rangle = ^1\text{S}_0$ and $|s\rangle = ^3\text{P}_0$ (g for ground and s for stable). Using a differential g factor [13], one can optically excite all $6I + 1$ individual $|g\rangle$ - $|s\rangle$ transitions in the presence of a magnetic field B [14] [see Fig. 1(a)]. Thus, any unitary operation on an individual register can be achieved [15,16].

To make single-register manipulation spatially selective, one can envision various strategies. The conceptually simplest strategy would adiabatically expand the lattice for the time of the manipulation, which can be done without changing the wavelength of the light using holographic techniques or angled beams [17]. Alternatively, we can make a temporary Raman transfer of a given pair of Zeeman levels of $|s\rangle$ up to $|s'\rangle = ^3\text{P}_2$ via $|e'\rangle = ^3\text{S}_1$ [Fig. 1(a)] [9]. One way to achieve spatial selectivity in this Raman transfer is to perform adiabatic passage, in which

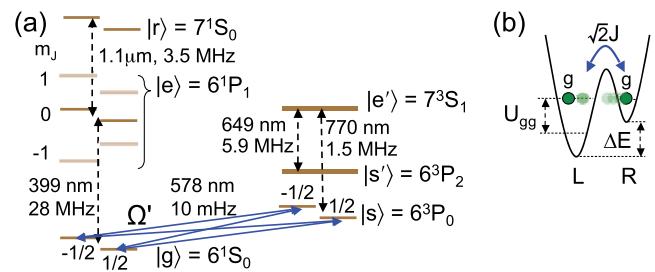


FIG. 1 (color online). (a) On the example of ^{171}Yb ($I = 1/2$) in the Paschen-Back regime for state $^1\text{P}_1$, relevant alkaline-earth-like level structure. (b) Interregister gate based on conditional resonant tunneling.

the beam on one of the transitions vanishes (and, thus, prohibits the transfer) at the location of one atom [18]. Subsequent manipulation of the two chosen Zeeman levels of $|s\rangle$ will then be automatically spatially selective. Any other pair of states within the register can be manipulated by swapping it in advance with the first one. Alternatively, one can use the $|s\rangle-|e\rangle-|s'\rangle$ Lambda system to achieve spatial selectivity even without state transfer by using dark-state-based selectivity [19].

Interregister gate.—We now assume that atoms are prepared in a Mott insulator state [20] in an optical lattice (or a fully polarized band insulator, which may be easier to load), with one atom per site. We isolate two adjacent atoms (left and right) using a superlattice [11,21]. We now show how to generate a two-qubit phase gate between the electronic qubits of these atoms (i.e., $|g, g\rangle \rightarrow -|g, g\rangle$) in a regime where the tunneling J is much smaller than the on-site interaction energy. As shown in Fig. 1(b), we bias the right well relative to the left well by a value ΔE equal to the interaction energy U_{gg} between two $|g\rangle$ atoms. If we had bosons with $I = 0$, then after time $\tau \sim 1/J$ the state $|g, g\rangle$ would pick up a minus sign due to resonant tunneling of the right atom to the left well and back. We now demonstrate how this gate works for fermions with arbitrary I . We consider the two-well single-band Hubbard Hamiltonian [22]

$$\begin{aligned} \hat{H}_h = & -J \sum_{\alpha, m} (\hat{c}_{L\alpha m}^\dagger \hat{c}_{R\alpha m} + \text{h.c.}) + \Delta E \sum_{\alpha, m} \hat{n}_{R\alpha m} \\ & - \sum_{i, \alpha, m} \mu_B m g_\alpha \hat{n}_{i\alpha m} + \sum_{i, \alpha, m < m'} U_{\alpha\alpha} \hat{n}_{i\alpha m} \hat{n}_{i\alpha m'} \\ & + V \sum_i \hat{n}_{ig} \hat{n}_{is} + V_{\text{ex}} \sum_{i, m, m'} \hat{c}_{igm}^\dagger \hat{c}_{ism'}^\dagger \hat{c}_{igm'} \hat{c}_{ism}. \quad (1) \end{aligned}$$

Here $i = L, R$ labels sites; $\alpha = g, s$; $m, m' = -I, \dots, I$; $\hat{n}_{i\alpha} = \sum_m \hat{n}_{i\alpha m}$; $\hat{n}_{i\alpha m} = \hat{c}_{i\alpha m}^\dagger \hat{c}_{i\alpha m}$, where $\hat{c}_{i\alpha m}^\dagger$ creates an atom in state αm on site i . The tunneling rate J and the bias ΔE are assumed for notational simplicity to be state independent. g_α is the g factor of state α . $V = (U_{sg}^+ + U_{sg}^-)/2$ and $V_{\text{ex}} = (U_{sg}^+ - U_{sg}^-)/2$ describe the “direct” and “exchange” interactions [23]. $U_{\alpha\beta}^x = (4\pi\hbar^2 a_{\alpha\beta}^x/M) \times \int d^3r |\phi_\alpha(\mathbf{r})|^2 |\phi_\beta(\mathbf{r})|^2$ [24], where M is atomic mass, $a_{\alpha\beta}^x = a_{gg}, a_{ss}, a_{sg}^+, a_{sg}^-$ are the four s -wave scattering lengths, ϕ_α are the Wannier orbitals. a_{sg}^- corresponds to the antisymmetric electronic state $|gs\rangle - |sg\rangle$ (implying a symmetric nuclear state), while a_{gg}, a_{ss} , and a_{sg}^+ correspond to the three symmetric electronic states (implying antisymmetric nuclear states). Since $|g\rangle$ and $|s\rangle$ have $J = 0$ and since hyperfine mixing of $|s\rangle$ with other states is small [13], we take $a_{\alpha\beta}^x$ to be independent of nuclear spin, which is consistent with experiments [14,25]. We note that the optical energy of $|s\rangle$ is absent in our rotating frame.

In our scheme, provided U_{gg} differs from other U 's, the interaction blockade [12] will prevent two atoms from

being on the same site unless they are both in state $|g\rangle$, so we can ignore all but the U_{gg} interaction terms. In this case, the Zeeman Hamiltonian can be rotated out. The first step of the gate is to increase the bias ΔE from 0 to U_{gg} for time $\tau = \pi/(\sqrt{2}J)$, and then set it back to 0. Defining $|g, g\rangle|m_2, m_1\rangle = \hat{c}_{Lgm_2}^\dagger \hat{c}_{Rgm_1}^\dagger |0\rangle$, this gives a 2π pulse between $I(2I+1)$ states $|g, g\rangle(|m_2, m_1\rangle - |m_1, m_2\rangle)$ ($m_1 < m_2$) and $\hat{c}_{Lgm_1}^\dagger \hat{c}_{Lgm_2}^\dagger |0\rangle$, so that the former pick up a factor -1 . The $I(2I+1)$ states that pick up the factor -1 are precisely all the $|g, g\rangle$ states with an antisymmetric nuclear state since two $|g\rangle$ atoms in a symmetric nuclear state cannot sit on one site. To make *all* $(2I+1)^2$ $|g, g\rangle$ states pick up the factor -1 , we require two more steps. In the second step, we apply a phase -1 on site R on all $|g, m\rangle$ with $m > 0$, repeat the bias, and repeat the phase. In the final step, we swap $|g, m\rangle$ and $|g, -m\rangle$ on site R , repeat the first two steps, and repeat the swap. This results in $|g, g\rangle \rightarrow -|g, g\rangle$ independent of the nuclear spin, i.e., a two-qubit phase gate on the two electronic qubits. All atom pairs in the superlattice that experience only the four biases are unaffected. Thus, together with spatially selective single-atom manipulation, this gate gives universal manipulation of the full lattice of quantum registers. The gate error due to virtual tunneling is $\sim (J/U)^2$, where J/U is the smallest relevant ratio of tunneling to interaction energy or to a difference of interaction energies. This error can be reduced if $|g\rangle$ and $|s\rangle$ lattices are independent [9]. Other errors in this gate are analogous to those studied in Ref. [12].

We now point out some advantages of this gate. The gate is essentially achieved by conditioning the resonant tunneling on the internal state of the atoms rather than on the number of atoms in the wells [11,12] or on the vibrational levels the atoms occupy [26]. Being resonant, the gate is faster ($\tau \sim 1/J$) than superexchange gates ($\tau \sim U/J^2$) [27]. At the same time, by conditioning the tunneling on the internal state, we avoid having two $|s\rangle$ atoms in one well [10], which may be subject to collisional losses. A key property of the gate is that it couples the electronic (communication) qubits without affecting the nuclear qubits. At the same time, a remarkable feature of our gate is that it would not have worked without the use of the nuclear degree of freedom, because two $|g\rangle$ atoms would not be able to sit on the same site in that case. This is in a sense the reverse of Ref. [8], where a gate on nuclear spins relies on the underlying electronic interactions. Finally, our gate can be easily extended to bosons. In particular, a single bias interval would suffice for bosons with two internal states $|g\rangle$ and $|s\rangle$ that have different interactions U_{gg} , U_{ss} , and U_{sg} (e.g., if $|g\rangle$ and $|s\rangle$ experience different potentials).

Electronic-qubit detection.—We now demonstrate the essential ability of our register to preserve all nuclear qubits during the fluorescence detection of the electronic qubit. The key ingredients will be off-resonant excitation [2] and/or a strong magnetic field [7]. The detection is

made by cycling the $|g\rangle\langle e| = ^1P_1$ transition (“e” for excited). To yield an error $p < 0.01$ after scattering $N \sim 100$ photons, the decay rate from $|e\rangle$ to $|g\rangle$ should exceed other decay rates from $|e\rangle$ by $>10^4$, which is typically satisfied [28,29]. We can thus restrict ourselves to a $4(2I + 1)$ -dimensional space describing the $|g\rangle\langle e|$ transition: $|g\rangle|m_I\rangle$ ($J = 0$) and $|e, m_J\rangle|m_I\rangle$ ($J = 1$). The Hamiltonian is then ($\hbar = 1$) [22]

$$\hat{H} = A\hat{\mathbf{I}} \cdot \hat{\mathbf{J}} + Q \frac{3(\hat{\mathbf{I}} \cdot \hat{\mathbf{J}})^2 + 3/2\hat{\mathbf{I}} \cdot \hat{\mathbf{J}} - K}{2IJ(2I-1)(2J-1)} + g_J\mu_B\hat{J}_zB - g_I\mu_N\hat{I}_zB - \Omega(|g\rangle\langle e, 0| + \text{h.c.}) - \Delta \sum_m |e, m\rangle\langle e, m|. \quad (2)$$

Here $K = I(I + 1)J(J + 1)$; A and Q ($Q = 0$ for $I = 1/2$) are the magnetic dipole and electric quadrupole hyperfine constants, respectively; g_J and $g_I = g_g$ are the relevant g factors; Ω and Δ are the Rabi frequency and the detuning of the π -polarized probe light. Using three Lindblad operators $\hat{L}_m = \sqrt{\Gamma}|g\rangle\langle e, m|$, the master equation is

$$\dot{\rho} = -i[\hat{H}, \rho] - \frac{1}{2} \sum_m (\hat{L}_m^\dagger \hat{L}_m \rho + \rho \hat{L}_m^\dagger \hat{L}_m - 2\hat{L}_m \rho \hat{L}_m^\dagger). \quad (3)$$

Two approaches to preserve nuclear coherence during fluorescence are possible. In the first one, a strong magnetic field ($g_J\mu_B B \gg A, Q$) decouples $\hat{\mathbf{I}}$ and $\hat{\mathbf{J}}$ in the Paschen-Back regime [7]. In the second one, a large detuning $\Delta \gg A, Q$ does the decoupling by means of the interference of Raman transitions via all excited states with a given $I_z + J_z$ [2]. Unless $Q \ll \Gamma$, the first approach fails because the frequencies of transitions $|g\rangle|m_I\rangle\langle e, 0|m_I\rangle$ differ by $\delta_{m_I} = 3Qm_I^2$ [see Fig. 2(a)]. However, when $Q \ll \Gamma$, the first approach may be preferable as it allows for much faster detection than the second (off-resonant) approach. While the two approaches can work separately, their combination is sometimes advantageous and allows for the following simple estimate of the nuclear decoherence due to off-resonant excitation. We assume $\Delta \gg Q$, Ω , Γ and a magnetic field large enough to decouple $\hat{\mathbf{I}}$ and $\hat{\mathbf{J}}$ (arbitrary B can be analyzed similarly). The number of photons scattered during time τ is then $N \approx \Gamma\tau\Omega^2/\Delta^2$.

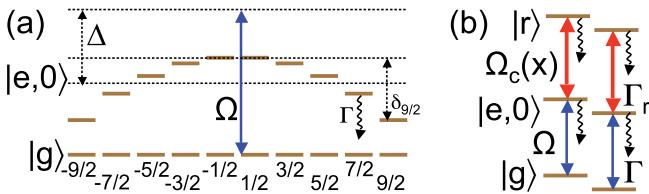


FIG. 2 (color online). Nuclear-spin-preserving electronic-qubit detection. (a) ^{87}Sr ($I = 9/2$) in the Paschen-Back regime (the shift $-g_I\mu_N m_I B$ not shown). (b) On the example of ^{171}Yb ($I = 1/2$), dark-state-based spatial selectivity [19].

Furthermore, for any two $m_I = m_1, m_2$, the four coherences $|g/e, 0\rangle|m_1\rangle - |g/e, 0\rangle|m_2\rangle$ form a closed system. Adiabatically eliminating the three coherences except for the ground one, the latter is found to decay with rate $\tilde{\Gamma}_{12} \approx (\delta_{m_1} - \delta_{m_2})^2\Omega^2\Gamma/(2\Delta^4)$, yielding an error $p \sim \tilde{\Gamma}_{12}\tau \sim N(Q/\Delta)^2$. Thus, to scatter $N = 100$ photons and obtain $p < 0.01$, we need $\Delta \gtrsim 100Q$.

To verify low p numerically, we use ^{171}Yb ($I = 1/2$) [30], ^{87}Sr ($I = 9/2$) [14], and ^{43}Ca ($I = 7/2$), for which $(\Gamma, A, Q)/(2\pi \text{ MHz}) = (28, -213, 0)$ [7], $(30.2, -3.4, 39)$ [7], and $(35, -15.5, -3.5)$ [31], respectively. Although less widely used, ^{43}Ca has the advantageous combination of small Q and large I . We prepare the atom in some state in the manifold $|g\rangle$, turn Ω on and off abruptly for a time τ , and then wait for all the population to decay down to $|g\rangle$, which transforms Eq. (3) into a superoperator $\hat{\mathcal{E}}$ acting on density matrices describing $|g\rangle$. Ideally, $\hat{\mathcal{E}}$ describes a unitary transformation \hat{U} that maps $|g\rangle|m\rangle \rightarrow \exp(i\phi_m)|g\rangle|m\rangle$ with $\phi_{-I} = 0$ and ϕ_m ($m > -I$) given by the phase of the diagonal elements of $\hat{\mathcal{E}}$ corresponding to the density matrix element $\rho_{m, -I}$. So $p \equiv 1 - \bar{F}$, where \bar{F} is the average gate fidelity of $\hat{\mathcal{E}}$ with respect to \hat{U} [32]:

$$\bar{F}(\hat{\mathcal{E}}, \hat{U}) \equiv \int d\psi \langle \psi | \hat{U}^\dagger \hat{\mathcal{E}}(|\psi\rangle\langle\psi|) \hat{U} | \psi \rangle. \quad (4)$$

We fix $N = 100$ and begin by considering the first approach (large B and $\Delta = 0$). In Yb, $B = 2T$ and $\Omega/2\pi = 30 \text{ MHz}$ ($\tau = 1.3 \mu\text{s}$) give $p \approx 0.01$. Since Yb has $I = 1/2$ (hence $Q = 0$), as $B \rightarrow \infty$, $p \rightarrow 0$: for example, $B = 10 \text{ T}$ and $\Omega/2\pi = 200 \text{ MHz}$ ($\tau = 1.1 \mu\text{s}$) give $p \approx 10^{-4}$. In Ca, small Q/Γ (≈ 0.1) also allows one to obtain high fidelity on resonance: $B = 1 \text{ T}$ and $\Omega/2\pi = 200 \text{ MHz}$ ($\tau = 0.9 \mu\text{s}$) give $p \approx 0.002$. Since Q is finite here, increasing B further does not reduce p to zero. Finally, in Sr, resonant scattering gives $p \gtrsim 0.1$ due to the large Q . Turning now to the second approach ($B = 0$ and large Δ), for Yb, Ca, and Sr, $(\Delta, \Omega)/(2\pi \text{ GHz}) = (15, 0.2)$, $(6, 0.07)$, and $(3, 0.04)$, respectively, give $\tau \sim 3 \text{ ms}$ and $p \approx 0.01$. An increase of Ω (to reduce τ) leads unfortunately to larger p at least partly due to the loss of adiabaticity in the evolution of coherences. The error can be reduced by further increasing Δ (or to some extent by decreasing Ω) and thus extending τ . We note that in this (second) approach, any probe light polarization can be used. Finally, the error can sometimes be significantly reduced by combining the two approaches. For example, adding $B = 2 \text{ T}$ to the above example of off-resonant detection in Ca yields $p \approx 4 \times 10^{-4}$. Depending on time constraints, available magnetic fields and laser power, as well as on the desired N and p , the parameters can be further optimized and adiabatic switching of Ω can be considered.

To make detection spatially selective, we can apply the dark-state-based single-site addressability [19], shown in Fig. 2(b) on the example of Yb in the Paschen-Back

regime. Let $|r\rangle$ be the second lowest 1S_0 state [Fig. 1(a)]. In addition to the probe laser Ω , we apply a spatially varying control field $\Omega_c(x)$ coupling $|e\rangle$ and $|r\rangle$ in two-photon resonance with Ω . If $\Omega_c(x)$ vanishes at the position of atom 1 and is strong on all the other atoms affected by Ω , only atom 1 will fluoresce, while all other atoms will be unaffected. For example, in the first Yb example above, application of $\Omega_c/2\pi = 1$ GHz reduces the number of scattered photons to $N \sim 0.01$ and gives only 1% decay of the $|g\rangle - |s\rangle$ coherence [33]. Alternatively, we can temporarily transfer, as described above, all Zeeman levels of $|s\rangle$ up to $|s'\rangle$ in all but one atom, apply a NOT gate on all electronic qubits, carry out the detection, and then undo the NOT gate and the Raman transfer. Finally, temporary lattice expansion and magnetic gradients [9] can also be used.

Conclusion.—We have shown how to implement and couple quantum registers based on individual alkaline-earth-metal-like atoms trapped in individual sites of an optical lattice. These quantum registers can be used as a starting point for fault-tolerant circuit-based quantum computation [3]. Alternatively, they can be used for high fidelity generation (and measurement) of two-colorable graph states [3,34], which include cluster states for the use in measurement-based quantum computation [35] and GHZ states for the use in precision measurements [36]. In particular, a cluster state can be generated in a highly parallel fashion [34] by first preparing all the electrons in state $|g\rangle + |s\rangle$ and then applying the two-qubit phase gate on each edge, which our scheme allows to do in 2 steps per each dimension of the lattice. We note that assuming high fidelity detection or a restricted error model, a four-qubit register ($I \geq 7/2$) is sufficient for the fault-tolerant operation of a quantum register [3]. However, even one ($I = 1/2$) or two ($I = 3/2$) extra qubits can be used to do simpler entanglement pumping and, thus, increase the fidelity of two-colorable-graph-state generation [34]. With its accessibility using current experimental techniques and with the possibility to convert the electronic qubits into flying qubits, our approach and its extensions to ions with similar internal structure should be immediately useful in fields such as precision measurements, quantum computation, and quantum communication.

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