Numerical Studies in Quantum Information Sciences

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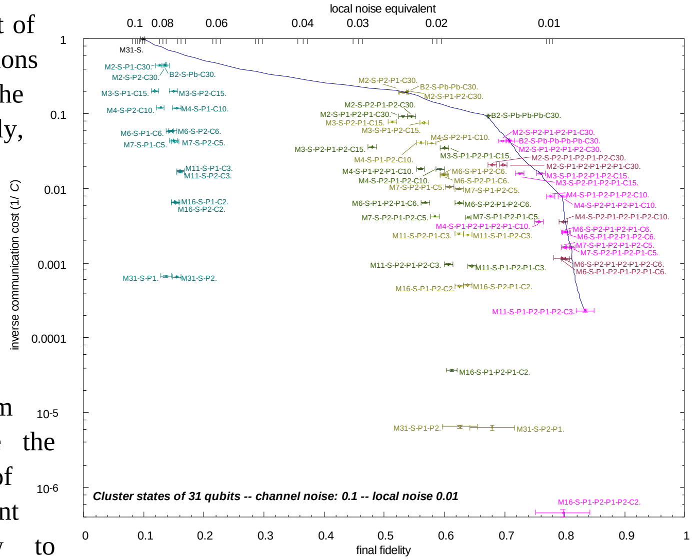
Quantum computers: With the ever increasing power of computers the range of problems that can be attacked with computational methods is growing larger and larger. Nevertheless, there are systems which seem to be beyond the possibility of numerical simulations. These are not only very large classical systems but also many systems governed by the laws of quantum mechanics. Due to the possibility of quantum-mechanical superpositions the amount of information needed to describe a quantum system grows exponentially with the system size, and hence, even small systems can already be intractable. In the early 1980s, Richard Feynman conjectured that a computer that took explicit advantage of the weird features of quantum mechanics may be capable of simulating quantum system that seem to be beyond the range of any other kind of quantum mechanics. In 1994, Peter Shor realized that such a device could also solve certain mathematical problems efficiently that are believed to be intractable with a classical computer, namely the problem of factorizing large integers.

A quantum computer would be a device very different from today's computer. Its "quantum bits" (qubits) have to be entities of microscopic size, single atoms or ions, or single electrons being held in some semiconductor structure. How to interact with such minute entities in a highly controlled manner, and how to shield them from unwanted influences, is currently being investigated by many research groups (including the groups of Professors Blatt, Zoller and Shnirman here in Innsbruck).

Quantum information science: Once one has succeeded in building a register of qubits and is able to write information to them, perform computational operations and read out their state, how can one use them? What else, except computation, might they be useful for? How can one protect the stored information against uncontrolled influences of the surrounding, and how to counter and undo such deterioration due to noise? To study these questions we may consider the qubits as abstract entities, forming a finite Hilbert space, and disregard mostly their physical nature. Nevertheless, studying the mathematical properties of this construction teaches a lot about physics. Most importantly, it is the ideal playground to study entanglement, i.e. the theory about correlations in nature. This project focuses on the research in this setting.

Numerics in quantum information science: As the very idea of a quantum computer is to access realms beyond the possibilities of ordinary computers it is obvious that we cannot simulate a quantum computer. We can, however, simulate certain aspects of it,



namely the effect of a subset of computational operations ("gates") that is known as the Clifford group. Fortunately, many important protocols in quantum information processing only require these gates. They include, most importantly, schemes for quantum error correction, which are essential to allow a quantum computer to work despite the unavoidable presence of noise, and for entanglement purification, which allow to refresh the entanglement between qubits after some of them have been transported over long distance (e.g., as photons through an optical fiber). While the effect of these protocols can be assessed analytically for simplified noise models, this gets difficult for more realistic models for the disturbances of the system. Here, the efficient simulatability of Clifford gate networks allows for numerical studies in Monte-Carlo style.

We have devised a highly efficient algorithm for such simulation and implemented it in C++ ("GraphSim" [1]). It allowed us to study in detail when a novel kind of entanglement purification protocol found by our group can outperform other methods [2]. The figure shows a result of simulations performed for this purpose. Each data point corresponds to one possible combination of the available strategies, the x axis shows the "quality" of the resulting purified quantum state and the y axis the resource required to obtain it.

Another focus of our research is the question whether the new insights that quantum information science has brought about entanglement may help to improve on simulation methods for spin systems (an often-used abstraction useful to model solid state matter for certain questions). We proposed a new variational technique, based on so-called weighted graph states and explored its viability in many numerical experiments [3, 4]. Currently, we are working on an attempt to combine our method with another technique, the so-called tensor-tree states.

For most of these computations, many runs are needed, either in order to get good Monte-Carlo statistics, or in order to explore a wide parameter range. This makes the use of a compute cluster necessary, and the University's facilities are hence of great use for our work.

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Selected publications:

[1] S. Anders, H. J. Briegel: *Fast simulation of stabilizer circuits using a graph state representation*. Physical Review A **73**, 022334 (2006).

[2] C. Kruszynska, S. Anders, W. Dür, H. J. Briegel: *Quantum communication cost of preparing multipartite entanglement.* Physical Review A **73**, 062328 (2006).

[3] S. Anders, M. B. Plenio, W. Dür, F. Verstraete, H. J. Briegel: *Ground state approximation for strongly interacting systems in arbitrary dimensions.* Physical Review Letters **97**, 107206 (2006)

[4] S. Anders, H. J. Briegel, W. Dür: *A variational method based on weighted graph states.* New Journal of Physics **9**, 361 (2007).

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