Influence of satellite motion on polarization qubits in a Space-Earth quantum communication link

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Abstract: In a Space quantum-cryptography experiment a satellite pointing system is needed to send single photons emitted by the source on the satellite to the polarization analysis apparatus on Earth. In this paper a simulation is presented regarding how the satellite pointing systems affect the polarization state of the single photons, to help designing a proper compensation system.

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References and links

7. R. Ursin et al., “Free-space distribution of entanglement and single photons over 144 Km,” quant-ph/0607182
1. Introduction

Free-space quantum key distribution is one of the most promising technologies in the fast evolving field of quantum communication. Several experiments have already been performed to demonstrate its feasibility by distributing single photons and entangled photon pairs at an increasing distance between the link ends [1-7]. Terrestrial free-space links suffer from light loss due mostly to objects interposed in the line of sight, beam distortion induced by atmospheric turbulence, bad weather conditions and aerosols and they are thus limited to rather short distance. A solution to this problem can be the use of Space and satellite technology [8-13]. Space-based links have the potential to realize global-scale quantum networking since they allow, in principle, a much larger propagation distance of photonic qubits compared to present fiber links. This is mainly due to the fact that most of the communication path is in empty Space, where the photons can freely propagate, and only a short section of the path is in atmosphere. The atmosphere provides low absorption in the regime of 600 nm - 850 nm (i.e. where good single-photon detectors are available) and is almost non-birefringent, which guarantees the preservation of photon polarization at a high degree. Several aspects must be taken into account in planning a Space-to-Earth link. In this paper we consider one of these aspects, namely the influence of the satellite motion on the transmission of polarization qubits, i.e. qubits encoded into the polarization state of single photons.

Polarization-encoding is currently one of the most widespread realizations of photonic qubits [14], which utilizes two orthogonal states of polarization to encode information onto the optical mode of a single photon. Since the atmosphere does not affect the polarization of photons, it is also the system of choice for free-space quantum communication schemes. In order to establish a quantum communication protocol between two parties a system is needed to send the single photons emitted by the source on the satellite to the ground station, where the receiver is located. This is achieved by a pointing system made of a set of mirrors; in this paper we consider a simplified model with two plane ones: the first, on the satellite, sends the photons to the ground station whatever the position of the satellite on the sky is while the second one, in the ground station, receives the photons and, wherever they come from, sends them to the polarization analysis apparatus.

A working quantum communication protocol involving single-photons necessitates a shared reference frame between the communicating parties. For the interesting case of a moving satellite (e.g. LEO or MEO) the problem arises that any spatial reference frame between a Space-born transmitter and an Earth-based receiver will also be modified due to the movement of the involved pointing and tracking mirrors. In the following, we provide an analysis of this effect and discuss possible compensation schemes such as the use of a reference beam. It is important to state that, differently from classical laser communication, in polarization-encoded quantum communication all the polarization states must be transmitted and received correctly, so that the compensation of polarization transformation induced by the channel is a crucial issue.
As long as polarization measurements involve only qubits, encoded in two orthogonal polarization states of a single photon, the classical theory of polarization and the quantum one coincide [15]; so, in the case under investigation, single photons emitted by a source on a satellite and detected by an Earth-based receiver, the classical Jones calculus will be used.

2. The model

To model this situation we chose a reference frame whose origin is the Earth center and the intersection between the Earth’s equatorial plane and the satellite’s orbital plane is y direction (see Fig. 1). The z direction is orthogonal to the equatorial plane, while the x direction is chosen so as to have an orthonormal frame. Let $\xi$ be the orbit inclination, that is the angle between the equatorial plane and the orbit plane.

To simplify the model we make the following approximations:

- the satellite orbit is supposed to be circular
- the model we propose takes into account only two pointing mirrors. A real system will be more complex than this and other optical devices could introduce their own perturbations to the polarization states of the transmitted photons. However, since fixed mirrors will provide only a constant offset to the polarization rotation, the time-dependent contribution will come only from the two pointing mirrors.
- the beam coming from the quantum communication source on the satellite is supposed to be tangent to the satellite’s orbit: this is certainly arbitrary. Moreover, in a real situation, the satellite could also rotate around its axis, changing the direction of the incoming beam.
- simulations are performed for aluminum mirrors

Fig. 1. Fixed reference frame: the origin is set in the Earth center, the z direction is orthogonal to the equatorial plane and the y direction is on the intersection between the equatorial and the orbital planes
The normal to the equatorial plane is: $\mathbf{N}_{\text{equat}} = (0,0,1)$ while the normal to the orbital plane can be obtained from this by a rotation of $\xi$ along the y axis:

$$\mathbf{N}_{\text{orb}} = \begin{bmatrix} \cos \xi & 0 & \sin \xi \\ 0 & 1 & 0 \\ -\sin \xi & 0 & \cos \xi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin \xi \\ 0 \\ \cos \xi \end{bmatrix}$$ (1)

The same procedure can be applied to find the equation of the satellite’s orbit. A circular orbit on the equatorial plane would have been described by: $\mathbf{x} = R_o (\cos \omega t, \sin \omega t, 0)$ where $R_o$ is the orbital radius. The actual orbit can be found from this performing a rotation of an angle $\xi$ around the y-axis:

$$\mathbf{x} = R_o (\cos \xi \cos \omega t, \sin \omega t, -\sin \xi \cos \omega t)$$ (2)

The position of the ground station on Earth is a point on a spherical surface of radius $R_e$:

$$\mathbf{M} = R_e (\cos \beta \cos \alpha, \cos \beta \sin \alpha, \sin \beta)$$ (3)

Since the Earth rotates around the z axis this angle varies in time: $\alpha = \alpha_0 + \omega T t$, where $\omega_T$ is the Earth’s angular velocity. On the ground station a different reference frame is needed, the frame which is natural for an observer on the station. This is determined by the orthonormal vectors:

$$\alpha_1 = \frac{\partial \mathbf{M}}{\partial \alpha} = (-\sin \alpha, \cos \alpha, 0)$$ (4)

$$\alpha_2 = \frac{\partial \mathbf{M}}{\partial \beta} = (-\sin \beta \cos \alpha, -\sin \beta \sin \alpha, \cos \beta)$$ (5)

$$\alpha_3 = \frac{\mathbf{M}}{||\mathbf{M}||} = (\cos \beta \cos \alpha, \cos \beta \sin \alpha, \sin \beta)$$ (6)

Note that, since the position of the ground station varies in time also the three vectors $\alpha_i$ vary in time.

2.1. Visibility of the satellite

A satellite is visible from a specific point on Earth’s surface only if the angle between the vector $\mathbf{v} = \mathbf{x} - \mathbf{M}$ and the normal to the Earth’s surface is between 0 and 90 degrees (see Fig. 2). That is:

- $\mathbf{M} \cdot \mathbf{v} \geq 0 \Rightarrow$ visible
- $\mathbf{M} \cdot \mathbf{v} < 0 \Rightarrow$ not visible

2.2. Reflection on the first mirror

We consider a light beam emitted by a source on the satellite that is reflected by a mirror (on the satellite) towards the receiving ground station. We arbitrarily assume that the initial beam direction is tangent to the satellite’s orbit, so, its propagation direction is:

$$\mathbf{L}_1 = (-\cos \xi \sin \omega t, \cos \omega t, \sin \xi \sin \omega t)$$ (7)

To completely describe this light beam we also need the direction of the s-polarization component; it is the normal to the orbital plane:

$$\mathbf{s}_0 = (\sin \xi, 0, \cos \xi)$$ (8)
This light beam hits the first mirror and, whatever the position \( \mathbf{x} \) of the satellite is, it is reflected down the ground station (identified by the vector \( \mathbf{M} \)). So, the reflected vector is:

\[
\mathbf{L}_2 = \frac{\mathbf{M} - \mathbf{x}}{r} \tag{9}
\]

where \( r = ||\mathbf{M} - \mathbf{x}|| \) is the distance between the satellite and the ground station.

The angle of incidence of the light beam on the mirror is:

\[
\theta_1 = \frac{1}{2} \arccos((\mathbf{L}_1) \cdot \mathbf{L}_2) \tag{10}
\]

The Jones matrix describing reflection on the mirror is:

\[
\begin{bmatrix}
    r_p(\lambda, \theta_1) & 0 \\
    0 & r_s(\lambda, \theta_1)
\end{bmatrix}
\tag{11}
\]

where \( r_p(\lambda, \theta_1) \) and \( r_s(\lambda, \theta_1) \) are the reflection coefficients of the mirror.

The reflected light has its on s-polarization direction, which is the normal to the incidence plane.

It can be thus calculated by:

\[
\mathbf{s}_1 = \frac{\mathbf{L}_1 \times \mathbf{L}_2}{||\mathbf{L}_1 \times \mathbf{L}_2||} \tag{12}
\]

The polarization of the light emitted by the source and of the light reflected by the first mirror are described in different reference frames, related to each other by a rotation of an angle \( \beta_{01} = \arccos(\mathbf{s}_0 \cdot \mathbf{s}_1) \)

This formula cannot tell us if the rotation is clockwise or counterclockwise, that is we cannot know from this if the angle \( \beta_{01} \) is positive or negative.

But from Fig. 3 we can see that the if \( 0 \leq \beta_{01} \leq \pi \) the vector \( \mathbf{s}_0 \times \mathbf{s}_1 \) is opposite to the direction of light propagation \( \mathbf{d} \), while if \( -\pi \leq \beta_{01} \leq 0 \) then the directions of \( \mathbf{s}_0 \times \mathbf{s}_1 \) and \( \mathbf{d} \) coincide. So we can define the quantity:

\[
\sigma_{01} = -\frac{\mathbf{s}_0 \times \mathbf{s}_1 \cdot \mathbf{L}_1}{||\mathbf{s}_0 \times \mathbf{s}_1 \cdot \mathbf{L}_1||} \tag{13}
\]
Fig. 3. Procedure to determine whether the rotation of the s-polarization direction is clockwise or counterclockwise

and we can calculate:

$$\beta_{01} = \sigma_{01} \arccos(s_0 \cdot s_1)$$  \hspace{1cm} (14)

This rotation is then described by the matrix:

$$\begin{bmatrix}
\cos \beta_{01} & \sin \beta_{01} \\
-\sin \beta_{01} & \cos \beta_{01}
\end{bmatrix}$$  \hspace{1cm} (15)

2.3. Reflection on the second mirror

Using a pointing mirror on the satellite the transmitted light is always sent towards the ground station, where it collected and sent to a polarization analyzer by means of a second pointing mirror.

The polarizer is taken to be horizontal with respect to ground, so its direction will be a linear combination of $\alpha_1$ and $\alpha_2$:

$$L_3 = (\cos \chi) \alpha_1 + (\sin \chi) \alpha_2$$  \hspace{1cm} (16)

So the laser beam, coming from the satellite in direction $L_2$, after reflection on the second mirror, goes in direction $L_3$. The subsequent analysis is carried on in the same way we made for the first mirror; we calculate the angle of incidence on the mirror $\theta_2$ and then the s-polarization direction as the normal to the second incidence plane:

$$s_2 = \frac{L_2 \times L_3}{||L_2 \times L_3||}$$  \hspace{1cm} (17)

Finally, proceeding as above above, we find the matrix:

$$\begin{bmatrix}
\cos \beta_{12} & \sin \beta_{12} \\
-\sin \beta_{12} & \cos \beta_{12}
\end{bmatrix}$$  \hspace{1cm} \beta_{12} = \sigma_{12} \arccos s_1 \cdot s_2$$  \hspace{1cm} (18)

Then the light must by analyzed by the polarizer, which has its own s-polarization direction, normal to the plane which describes the ground in the station; that is:

$$s_3 = \frac{\alpha_1 \times \alpha_2}{||\alpha_1 \times \alpha_2||}$$  \hspace{1cm} (19)

So we need a final rotation of an angle:

$$\beta_{23} = \sigma_{23} \arccos s_2 \cdot s_3$$  \hspace{1cm} (20)

performed by the matrix:

$$\begin{bmatrix}
\cos \beta_{23} & \sin \beta_{23} \\
-\sin \beta_{23} & \cos \beta_{23}
\end{bmatrix}$$  \hspace{1cm} (21)
2.4. Total polarization state

The final polarization state is described by:

\[
\begin{bmatrix}
E'_p \\
E'_s
\end{bmatrix} = \begin{bmatrix}
\cos \beta_{23} & \sin \beta_{23} \\
-sin \beta_{23} & \cos \beta_{23}
\end{bmatrix} \begin{bmatrix}
r_p(\lambda, \theta_2) & 0 \\
0 & r_s(\lambda, \theta_2)
\end{bmatrix} \begin{bmatrix}
\cos \beta_{12} & \sin \beta_{12} \\
-sin \beta_{12} & \cos \beta_{12}
\end{bmatrix} \begin{bmatrix}
r_p(\lambda, \theta_1) & 0 \\
0 & r_s(\lambda, \theta_1)
\end{bmatrix} \begin{bmatrix}
\cos \beta_{01} & \sin \beta_{01} \\
-sin \beta_{01} & \cos \beta_{01}
\end{bmatrix}
\] (22)

From the final polarization state we can find the normalized Jones vector:

\[
\frac{1}{\sqrt{|E'_p|^2 + |E'_s|^2}} \begin{bmatrix}
E'_p \\
E'_s
\end{bmatrix}
\] (23)

3. Simulations

Using our model, described in equation (22), we performed simulations for the satellite LA-GEOS 2, which has the following parameters:

- inclination = 52.68 degrees
- period = 222.6 minutes
- axes = 5616 Km * 5951 Km
- orbit eccentricity = 0.0135

We considered a latitude of 37 degrees for the ground station, which corresponds to a location in Southern Europe. In our simulations we used aluminum mirrors, whose refractive index, at several wavelengths of interest, is [16] :

- \( n(\lambda_1) = 0.129 + 3.25i \) (\( \lambda_1 = 532\text{nm} \)) \( n(\lambda_2) = 0.144 + 5.3i \) (\( \lambda_2 = 810\text{nm} \))
- \( n(\lambda_3) = 0.23 + 7.1i \) (\( \lambda_3 = 1064\text{nm} \)) \( n(\lambda_4) = 0.45 + 9i \) (\( \lambda_4 = 1500\text{nm} \))

In the case of a mirror made by a simple metallic surface the reflection coefficients are the Fresnel coefficients [17]:

\[
\begin{align*}
r_p(\lambda, \theta_i) &= \frac{n_o(\lambda) \cos \theta_i - n(\lambda) \cos \theta_i}{n_o(\lambda) \cos \theta_i + n(\lambda) \cos \theta_i} \\
r_s(\lambda, \theta_i) &= \frac{n_o(\lambda) \cos \theta_i - n(\lambda) \cos \theta_i}{n_o(\lambda) \cos \theta_i + n(\lambda) \cos \theta_i}
\end{align*}
\] (24)

where \( n_o(\lambda) \) is the refractive index of air, \( n(\lambda) \) is the refractive index of the metal surface of the mirror, \( \theta_i \) is the incidence angle on the mirror and \( \theta_f \) is such that: \( n_o(\lambda) \sin \theta_i = n(\lambda) \sin \theta_f \).

In Fig. 4 the satellite trajectories in the ground station reference frame are presented, it is evident how the satellite comes and goes along different path, with mirrors’ angles changing according to the parameters of each single passage. In Fig. 5 the Poincaré spheres for two different satellite passages are plotted: the polarization change is evident. Moreover different wavelengths behave in different ways and it is not possible to find a wavelength which minimizes the perturbations. Finally we calculated the Stokes parameters for 3000 satellite passages at \( \lambda = 810 \text{nm} \) and we plotted them in Fig. 6 to have a statistical understanding of polarization changes: we can see that the polarization state can be located almost anywhere on the Poincaré sphere with a higher probability on a strip around the equatorial plane.
Fig. 4. Different passages of the satellite above the ground station (represented by the GS in the centre of the XY plane). The satellite comes along different trajectories and so the pointing mirrors must be tilted in order to send the photons to the ground station, whatever the position of the satellite is. This makes the reference frame of the satellite rotate in respect to the reference frame on the ground station, and changes the angle of incidence on the mirrors, resulting on a modification of the polarization states of the emitted photons.

Fig. 5. Poincaré spheres showing the received polarization states for two different satellite passages on the sky and four different photon wavelengths. The source on the satellite emits a horizontally-polarized photon, whose polarization state, due to rotation of the reference frames determined by the satellite motion and to reflection on mirrors, is in general different from the emitted one and changes in time. Moreover, the polarization states of photons of different wavelengths change in different ways, because of the different responses of mirrors. Elliptical polarization states can be due to complex refractive indices of the mirrors. In particular this result also indicates that it is difficult to use a reference laser at a different wavelength for polarization compensation.
4. Conclusions

Even though realistic optical systems for satellite quantum communication could be more complex than our simple two-mirror model, most proposed schemes are based on a fixed telescope and a movable steering mirror for fine pointing at each communication side. Since the effect of fixed mirrors is just to provide a constant offset, they will just introduce time-independent modifications which can be compensated by proper calibration.

Besides instrumental contributions, also the atmosphere could introduce perturbations on the polarization of an electromagnetic wave, due to turbulence, scattering processes or Faraday effect. According to theoretical models and experimental data [18, 19], turbulence could give a rotation of the order of $10^{-11}\text{ rad/Km}$, while scattering could contribute with a rotation of about $10^{-4}\text{ rad/Km}$; both are much smaller than the what we have found in our simulations for the tracking system. A simple calculation shows that also Faraday effect is neglectable. Assuming for the magnetic field $B$ its maximum value at the Earth surface ($B = 60\mu T$), for the atmosphere depth a value of $d = 10\text{ Km}$ and for the Verdet constant of air the value [20] $V = 6.83 * 10^{-6}\text{ min/(Gcm)} = 1.9 * 10^{-5}\text{ rad/(T*m)}$ we find that the plane of polarization undergoes a rotation of an angle:

$$\chi = VBd = 0.001\text{ rad}$$

This value, which can be considered as a higher bound since we assumed the geomagnetic...
field constant at its value at the Earth's surface (while it decreases with $r^{-3}$ moving away into the atmosphere) is clearly unimportant.

From our analysis, it is evident that the moving mirrors of the satellite’s and ground station’s pointing system are the most important source of perturbation in a quantum communication link between Space and Earth, introducing a significant time-dependent rotation of the polarization states of the signal photons. This result shows that it is necessary to use an active compensation system, since for quantum communication all the polarization states must be transmitted and received correctly.

A possible implementation of a compensation system could be by means of a reference laser beam whose wavelength is different from the one of the signal photons. In the ground station one could perform a polarization analysis of the reference beam and evaluate the transformation that has been performed on it, and then he could apply the inverse transformation on the signal photons polarization states. The problem is that, as shown in Fig. 5, the transformations induced on different wavelengths are not identical, and therefore this kind of compensation could not be perfect.

Another possible approach could be to use the same wavelength for signal and reference photons, but alternating the times in which they are emitted, so as to implement a time-multiplexing configuration. This scheme, provided that the transmission and analysis of the reference beam is repeated at a sufficiently high rate required to keep up with the temporal variations of the polarization properties, will allow near perfect compensation, but will slow down the quantum communication, such as the rate of key exchange in a quantum cryptographic link.

A third approach is to deterministically calculate the actual polarization rotation in real time, given the satellite trajectory and the pointing angles are known, and accordingly vary the polarization compensation rotation at the ground station as the satellite passes by. However, this approach requires that the refractive indices of all the mirrors in the system are well known, are stable against physical influences such as temperature fluctuations, and show a long-term stability.

Our results are crucial for Space-based quantum communication, since any scheme for realizing single-photon polarization-encoded quantum communication or quantum key distribution, will have to utilize a configuration such as, or similar to, this geometry.

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