applied uniformly on all ions. We want the phase-space-displacement dipole forces on all ions (in the same state) to be the same, which is the case in a one-dimensional (or two-dimensional) array of trapped ions, if the forces act on a center-of-mass mode whose motion is perpendicular to the ion string (or plane) (31). The readout is robust against noise because the observable is a two-state “all-on” versus “all-off” signal. Despite experimental imperfections, our demonstration of the method on three ions shows a gain of 45% over that of a perfect Ramsey experiment with non-entangled states.

During the detection period, we obtained on average 0.2 counts after preparing the state $|\uparrow\rangle$, 30 counts after preparing the state $|\downarrow\rangle$, 30 counts after preparing the state $|\downarrow\rangle$. If the number of counts after a spectroscopy experiment was below 15, we assumed the outcome to be $|\downarrow\rangle, 30$; otherwise, we assumed the outcome to be $|\downarrow\rangle, 30$. The fringes resulting from averaging 1000 such experiments show a sinusoidal oscillation with period length 2$\pi/3$ (Fig. 2). The fringe contrast (0.84(1)) yields a phase sensitivity that is 0.84 × $\sqrt{3} = 1.45(2)$ as high as that of a perfect Ramsey experiment with three unentangled particles. With perfect operations and readout, the method would yield a fringe contrast equal to 1 and thus reach the Heisenberg limit, giving an increase of phase sensitivity of $\sqrt{3} = \sqrt{N}$. The gain obtained by using GHZ states can be offset by the corresponding faster decoherence if the free precession time is comparable to the dephasing time (28). However, the Ramsey time is often limited by other practical considerations, such as the desire to servo an oscillator to an atomic transition in a time that is much shorter than the decoherence time or by limitations imposed by the spectrum of the oscillator that drives the transitions (29, 30). In those cases, the entanglement provided by generalized GHZ states gives the expected gain.

We prepared a GHZ state of three atomic ions with a fidelity of 0.89(3). We used this state to demonstrate a new method for precision spectroscopy on entangled particles that can reach the Heisenberg limit and is of practical use because of its immunity to errors in final state detection. Because only collective ensemble preparation and detection pulses are used, the extension of the method to larger numbers of particles (N) is straightforward. For large values of N, the required interaction proportional to $J^2$ can be implemented with either the approach described in (10) or by further generalization of the phase gate because the operators $e^{i\alpha N}$ and $e^{i\alpha^2/2}$ are equivalent up to a rotation.
qubits has been demonstrated (7–13), our experiment allows the deterministic generation of three-qubit entangled states and the selective readout of an individual qubit followed by local quantum operations conditioned on the readout.

The experiments are performed in an elementary ion-trap quantum processor (14, 15). For the investigation of tripartite entanglement (16–18), we trap three $^{40}$Ca$^+$ ions in a linear Paul trap. Qubits are encoded in a superposition of the S$_{1/2}$ ground state and the metastable D$_{5/2}$ state (lifetime $\tau = 1.16$ s). Each ion-qubit is individually manipulated by a series of laser pulses on the $S = S_{1/2}(m_j = -1/2)$ to $D = D_{5/2}(m_j = -1/2)$ quadrupole transition near 729 nm, with the use of narrow-band laser radiation tightly focused onto individual ions in the string. The entire quantum register is prepared by Doppler cooling, followed by sideband ground-state cooling of the center-of-mass vibrational mode ($\omega = 2\pi \times 1.2$ MHz). The ions' electronic qubit states are initialized in the S state by optical pumping.

Three qubits can be entangled in only two inequivalent ways, represented by the Greenberger-Horne-Zeilinger (GHZ) state, $|\text{GHZ}\rangle = (|SSS\rangle + |DDD\rangle)/\sqrt{2}$, and the W state, $|\text{W}\rangle = (|DDS\rangle + |DSD\rangle + |SSD\rangle)/\sqrt{3}$ (17). The W state can retain bipartite entanglement when any one of the three qubits is measured in the $|\{S, D\}\rangle$ basis, whereas for the maximally entangled GHZ state a measurement of any one qubit destroys the entanglement. We synthesize the GHZ state with a sequence of 10 laser pulses and the W state with a sequence of five laser pulses (19).

Full information on the three-ion entangled states is obtained by state tomography (20, 21) using a charge-coupled device camera for the individual detection of ions. The pulse sequences generate three-ion entangled states within less than 1 ms. Determining all 64 entries of the density matrix with an uncertainty of less than 2% requires about 5000 experiments, corresponding to 200 s of measurement time.

In the experimental results for the absolute values of the density matrix elements of GHZ and W states, $\rho_{\text{GHZ}}$ and $\rho_{\text{W}}$ (Fig. 1, A and B), the off-diagonal elements are observed with heights nearly equal to those of the corresponding diagonal elements and with the correct phases (19). Fidelities of 72% for $\rho_{\text{GHZ}}$ and 83% for $\rho_{\text{W}}$ are obtained. The fidelity is defined as $|\langle \Psi_{\text{ideal}} | \rho_{\text{exp}} | \Psi_{\text{ideal}} \rangle|$, where $\Psi_{\text{ideal}}$ denotes the ideal quantum state and $\rho_{\text{exp}}$ is the experimentally determined density matrix. All sources of imperfection have been investigated independently (14) and the measured fidelities are consistent with the known error budget. Note that for the W state, coherence times greater than 200 ms were measured (exceeding the synthesis time by almost three orders of magnitude), whereas for the GHZ state only times of $\sim 1$ ms were found. This is because the W states are a superposition of three states with the same energy. Thus, the dephasing due to magnetic field fluctuations is much reduced in contrast to a GHZ state that is maximally sensitive to such perturbations. Similar behavior has been observed with Bell states (21, 22).

Having tripartite entangled states available as a resource, we make use of individual addressing to read out only one of the three ions' quantum state while preserving the coherence of the other two. Qubits are protected from being measured by transferring their quantum information into superpositions of levels that are not affected by the detection—that is, by a light-scattering process. In $\text{Ca}^+$, an additional Zeeman level $D' = D_{5/2}(m = -5/2)$ can be used for this purpose. Thus, after the state synthesis, we apply two $\pi$-pulses on the S-D' transition of ions 2 and 3, moving any S population of these ions into their respective D' level. The D and D' levels do not couple to the detection light at 397 nm (Fig. 2). Therefore, ion 1 can be read out by the electron shelving method as usual (15). After the selective readout, a second set of $\pi$-pulses on the D'-S transition transfers the quantum information back into the original computational subspace $|S, D\rangle$.

For a demonstration of this method, GHZ and W states are generated and qubits 2 and 3 are mapped onto the $|D, D'\rangle$ subspace. Then, the state of ion 1 is projected onto S or D by scattering photons for a few microseconds on the S-P transition. In a first series of experiments, we did not distinguish whether ion 1 was projected into S or D. After remapping qubits 2 and 3 to the original subspace $|S, D\rangle$, the tomography procedure is applied to obtain the full density matrix of the resulting three-ion state. As shown in Fig. 1C, the GHZ state is completely destroyed; that is, it is projected into a mixture of $|SSS\rangle$ and $|DDD\rangle$. In

**Fig. 1.** (A) Absolute values of the density matrix elements of the experimentally obtained GHZ quantum state. The off-diagonal elements for SSS and DDD indicate the quantum correlation clearly. (B) Absolute values of the density matrix of the W state. Off-diagonal elements and diagonal elements are at equal height. (C) GHZ state after measuring the first qubit only. The GHZ state coherences have fully disappeared as compared with the results shown in (A). The state is thus fully described by a classical mixture. (D) W state after measuring the first qubit. Only the coherences involving the first qubit have disappeared, whereas two-ion Bell-type entanglement persists between the second and the third qubit. The state thus contains quantum correlations even after a local projective measurement.

**Fig. 2.** Selective readout of ion 1: Ions 2 and 3 are protected from measurement by transfer into dark states. Only the relevant levels of the three $\text{Ca}^+$ ions are shown.
The GHZ state can be used to deterministically transform tripartite entanglement into bipartite entanglement by means of only local operations and one-qubit operations. For this, we first generate the GHZ state \((\sqrt{2}SDS)/\sqrt{2}\). In a second step, we apply a \(\pi/2\) pulse to ion 1, with phase \(3\pi/2\), rotating a state \(|S\rangle\) to \(|(S)\rangle/\sqrt{2}\) and \(|D\rangle\) to \(|(S) + |D\rangle\rangle/\sqrt{2}\), respectively. The resulting state of the three ions is \((|D)(SD) + |DS\rangle + |S)(SD) + |DS\rangle)/\sqrt{2}\). A measurement of the first ion, resulting in \(|D\rangle\) or \(|S\rangle\), projects qubits 2 and 3 onto the state \((|SD) - |DS\rangle)/\sqrt{2}\) or the state \((|SD) + |DS\rangle)/\sqrt{2}\), respectively. The corresponding density matrix is plotted in Fig. 4A. With the information on the state of ion 1 available, we can now transform this mixed state into the pure state \(|S)(SD) + |DS\rangle)/\sqrt{2}\) by only local operations. Provided that ion 1 is found in \(|D\rangle\), we perform an appropriate rotation (19) on ion 2 to obtain \(|D)(SD) + |DS\rangle)/\sqrt{2}\). In addition, we flip the state of ion 1 to reset it to \(|S\rangle\). Figure 4B shows that the bipartite entangled state \(|S)(SD) + |DS\rangle)/\sqrt{2}\) is produced with fidelity of 75% (19). This procedure can also be regarded as an implementation of a three-spin quantum eraser, as proposed in (27).

To find out if the qubit is in \(|D\rangle\) in 65 ± 2% of the cases. Fig. 3. (A) Histogram of photon counts within 500 \(\mu s\) for ion 1 and threshold setting. (B and C) Density matrix of ions 2 and 3 conditioned on the previously determined quantum state of ion 1. The absolute values of the reduced density matrix are plotted for ion 1 measured in the \(|S\rangle\) state (B) and ion 1 measured in the \(|D\rangle\) state (C). Off-diagonal elements in (B) show the remaining coherences.

Fig. 4. (A) Real part of the density matrix elements of the system after ion 1 of the GHZ state \((\sqrt{2}SDS)/\sqrt{2}\) has been measured in a rotated basis. (B) Transformation of the GHZ state \((\sqrt{2}SDS)/\sqrt{2}\) into the bipartite entangled state \(|S)(SD) + |DS\rangle)/\sqrt{2}\) by conditional local operations. Note the different vertical scaling of (A) and (B).

Our results show that selectively reading out a qubit of the quantum register indeed leaves the entanglement of all other qubits in the register untouched. Even after such a measurement has taken place, single-qubit rotations can be performed with high fidelity. Such techniques represent a step toward the one-way quantum computer (24). The implementation of unitary transformations conditioned on measurement results allows for the realization of deterministic quantum teleportation and of active quantum error correction algorithms. With further improvements of the gate fidelity, it will be possible to realize two different protocols for quantum error correction—either to perform a strong measurement on the ancilla qubit and to rotate the target qubit correspondingly or, alternatively, to rotate the target qubit conditional upon the still-unknown ancilla quantum state by conditional gate operations and to reset the ancilla afterward. It remains to be determined which of the two methods is favorable for our setup.

References and Notes
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Supporting Online Material
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Materials and Methods
Tables S1 to S3
References
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