There are at least three different ways in which quantum physics is connected with the concept of information. One is the relationship between quantum interference and knowledge. This was at the very heart of the early debates concerning the meaning of quantum mechanics, most notably the Bohr–Einstein dialogue [1]. That debate concerned the problem which occurred when quantum mechanics came up against the hitherto accepted notion that physics must describe reality as directly as possible and in an unambiguous and complete way. The debate was resolved by the Copenhagen interpretation in the most radical, conceptually challenging and foresightful manner, although for many physicists today, the Copenhagen interpretation is still conceptually unacceptable.

The second connection between quantum physics and information was the discovery in the early 1990s that quantum concepts could be used for communication and for processing information in completely novel ways. These include such topics as quantum cryptography, quantum teleportation and quantum computation [2].

The third connection between quantum physics and information has been emerging gradually over the last few years with the conceptual groundwork for this connection going back to the works of von Weizsaecker [3] and Wheeler [4]. It is the notion that information is the basic concept of quantum physics itself. That is, quantum physics is only indirectly a science of reality but more immediately a science of knowledge.

The present paper will touch upon all three connections between quantum physics and information.

### 3.1 Information and Quantum Interference

The connection between quantum interference and information is best illustrated by the double slit experiment (Fig. 3.1). This experiment already poses the challenging question: Does quantum mechanics describe reality or information? If we briefly consider the experiment with electrons – under what conditions do interference fringes arise at the observation plane?

Such fringes can easily be understood on the basis of interference of waves passing through both slits. Yet, as soon as we perform the experiment with
single particles, as has been done with photons, electrons, neutrons, atoms and molecules, the question arises: How does an individual particle which, one would naturally expect, has to pass through either slit, know whether or not the other slit is open? Richard Feynman [5] wrote for the double-slit experiment: “In reality, it contains the only mystery.” The modern Copenhagen way to talk about these questions is to assume that it only makes sense to talk about a property of a system if one actually cares to determine it or if at least the possibility for determining it exists. Or, in an even more modern way, the interference fringes arise if and only if there is no possibility, not even in principle, to determine which path the particle took. And, most importantly, it is not relevant whether or not we care to take note of that information. All that is necessary is whether or not the information is present somewhere in the universe. Only if such information is not present do interference fringes occur.

Indeed, the most interesting situations arise if the path information is present at some point in time, but deleted or erased in an irreducible way later on. Then, as soon as that information is irreversibly deleted, the interference fringes can occur again. Here, it is important to note that the mere diffusion of the information into larger systems, maybe even as large as the whole universe, is not enough to destroy the information. As long as it is there, no matter how well hidden or how dispersed, the interference fringes cannot occur.
Fig. 3.2. Two mutually complementary arrangements of the Heisenberg microscope. A photon is scattered from an electron and then enters the Heisenberg microscope. If a detector (or an observation screen) is placed in the imaging plane of the Heisenberg microscope lens, it can reveal the path the electron takes through the slit assembly, which therefore cannot show an interference pattern, assuming that it passes through an interferometer. On the other hand, if the detector is placed in the focal plane of the lens, it projects the state of the electron into a momentum eigenstate which cannot reveal any position information and therefore no information about which slit the electron has passed through. Interference fringes may thus occur.

This feature is most interestingly shown in a gedanken experiment where one combines the famous Heisenberg microscope with an electron double slit experiment.

We consider an electron interferometer where we may, if we decide to do so, determine the path the electron takes. This is done by scattering photons from the electrons passing through the double slit assembly. Clearly, these photons can be used to determine the path the electrons take by finding out in which slit they have been scattered (assuming that their wavelength is short enough). The simplest way to determine that position of scattering is to use the Heisenberg microscope as shown in Fig. 3.2. One can simply put a position-sensitive photon detector into the image plane and then, depending on where one observes the photon, one knows the path taken by the electrons. Therefore, no interference fringes can occur in that situation.

One might say now that one could simply not determine the position by not putting a photon detector into the image plane. Then one does not obtain information about the path taken, and one might be tempted to argue that interference fringes should occur. Yet the scattered photons nevertheless carry away the information about where they have been scattered and the path taken could be determined at an arbitrary time. Therefore, even if one does not care to read out this information, interference fringes should not arise.
Indeed, one could imagine that someone in a distant galaxy, equipped with very advanced technology, collects enough of the probability wave of the photon scattered and is thus able to determine the path taken. Therefore, even if one does not look at the scattered photon, no interference fringes should arise for the electron as long as the photon carries the path information.

In order to obtain interference fringes, one has to erase the information carried by the photon in an irrevocable way. That can best be done by detecting the photon, not in the image plane, but in the focal plane of the lens. Recalling the basic concept of Fourier optics, we realize that a point in the focal plane of a lens corresponds to an incoming momentum (or direction) on the other side of the lens. Thus it follows that registration of the photon in the focal plane projects the state of the scattered photon onto a momentum eigenstate which does not contain any position information. Therefore, once the photon is registered in the focal plane, all position information is gone and the corresponding electron interferes with itself.

This experiment has actually been realized, not using an electron and a photon, but using two photons exploiting the notion of entanglement [6, 7]. In this experiment (Fig. 3.3), one creates pairs of momentum entangled photons. One of the two photons plays the role of the electron and passes through a double slit assembly. That photon is registered behind the double slit at the double slit detector. The other photon plays the role of the scattered photon in the Heisenberg microscope experiment. It passes through the Heisenberg lens and then to the Heisenberg detector. Because of the strong entanglement between the two photons, the photon passing through the double slit does not show any interference pattern. In fact, the photon passing through the Heisenberg lens can be used to determine the path taken by the photon passing through the double slit. This is done by placing the detector in the image plane of the lens.

Alternatively, if one places the Heisenberg detector in the focal plane, the incoming photon and therefore also the entangled photon are both projected onto momentum eigenstates and the double slit interference fringes arise for photons observed in coincidence with a registration at the Heisenberg detector (see Figs. 3.3 and 3.4).

In this experiment, we also notice an interesting and elegant feature, viz., the low count rate. The peak count rate in the double slit pattern in Fig. 3.4 is about 120 photons in 60 seconds. This means that it is absolutely beyond doubt that the interference pattern is built up individual photon by individual photon.

These experiments can be seen as a confirmation of the viewpoint that it does not make sense to assign any property to a physical system irrespective of observation. In our case, the property as to whether the photon passing the double slit assembly can be seen as a particle or as a wave depends on what happens to the first photon. And this may actually occur at a time after the photon passing through the double slit assembly has already been registered!
Fig. 3.3. Double slit experiment for a photon of an entangled pair [6, 7]. A pair of momentum-entangled photons is produced in the crystal by type-I parametric down-conversion. One of the photons enters the Heisenberg microscope and is detected by the Heisenberg detector placed behind the Heisenberg lens. (It plays the role of the $\gamma$-quantum in the standard Heisenberg microscope experiment.) The other photon enters the double slit assembly and is detected by the double slit detector. (It plays the role of the electron.) If the Heisenberg detector is placed in the imaging plane of the lens, it can reveal the path the other photon takes through the slit assembly, which therefore cannot show interference. Alternatively, if the Heisenberg detector is placed in the focal plane of the lens, it projects the state of the other photon into a momentum eigenstate which cannot reveal any information about the slit the photon passes through. This photon therefore exhibits an interference pattern in coincidence with the registration of the other photon in the focal plane of the Heisenberg lens.

One might view this as a nice corroboration of Niels Bohr's famous dictum: "No phenomenon is a phenomenon unless it is an observed phenomenon."

These experiments also shed interesting light on the role of the observer with respect to reality. We note that it is the experimentalist who chooses the apparatus. The experimentalist, in our case Birgit, decides whether to put the detector into the focal plane or, say, into the image plane. That way, she determines which property of the system, wave or particle, can be reality. We might thus conclude that the experimentalist choosing the apparatus determines which physical quantity, i.e., quality, can be reality. In that sense, the experimentalist's choice is constitutive of the universe. However, the specific outcome here, which of the two slits the particle passes through in one case or where on the observation plane it arrives in the other, cannot be influenced by her. That way, Nature avoids complete controllability by the observer.
Fig. 3.4. Two mutually exclusive patterns registered by the double slit detector placed behind the double slit assembly (Fig. 3.3) as a function of its lateral position. The graphs show counts registered by that detector in coincidence with the registration at the Heisenberg detector if it is placed in the imaging plane of the lens (upper) and if it is placed in the focal plane of the lens (lower). Only in the latter case do the counts exhibit an interference pattern as the observation at the Heisenberg detector does not reveal the path the photon takes through the double slit assembly. Note the low intensity which indicates that the interference pattern is built up by individual photons, one at a time.

3.2 Towards a Quantum Information Technology

Unexpectedly for many, experiments motivated by fundamental and philosophical concerns have led to novel concepts for processing information. Quantum communication and quantum computation are the two areas where such new protocols in information technology have been developed over the last few years [2]. Interestingly, these new concepts and protocols rely on three fundamental notions. These are:

- the randomness of the individual measurement outcome,
- quantum complementarity,
- quantum entanglement.

A scheme which uses all three concepts together is entanglement-based quantum cryptography [8]. Let us therefore briefly discuss the essential point of the protocol without going into too much detail.

Let us assume that Alice and Bob share the entangled state of two qubits
Fig. 3.5. Encryption of an image of the Venus of Willendorf in the experimental realization of quantum cryptography [11]. The image is encrypted by Alice via bitwise XOR operation with her key. She transmits the encrypted image (b) to Bob via the computer network. Bob decrypts the image with his key, resulting in (c), which shows only a few errors due to the remaining bit errors in the keys

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B).$$ \hspace{1cm} (3.1)

Here a qubit is a two-state quantum system, the base states being denoted as |0\rangle and |1\rangle corresponding to the bit values 0 and 1, respectively. The state (3.1) is one of the maximally entangled (Bell) states [9]. The qubit A is held by Alice and qubit B by Bob after they have been produced somehow in the entangled state. It is now evident that, if Alice and Bob both perform measurements in the \{|0\rangle, |1\rangle\} basis, they will obtain the same random result 0 or 1 on their qubits. Hence, after having measured many pairs, the two arrive at identical sequences of random numbers. These numbers can be used as keys to encode information. It has been known since Vernam [10] that such keys are secure under two conditions: firstly, that they are used only once (one-time-pad) and secondly, that they are completely random. The procedure to detect an eavesdropper is to switch randomly between two bases, the computational one \{|0\rangle, |1\rangle\} and the complementary one, often also referred to as conjugate:

$$|0'\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle), \quad |1'\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle).$$ \hspace{1cm} (3.2)

In the conjugate basis the entangled photon has the same mathematical form as (3.1). Alice and Bob will switch randomly and independently between the two bases. Evidently they obtain the same bit value whenever they happen to have the same basis. An eavesdropper, Eve, somewhere on the line has to guess which basis Alice and Bob chose. Clearly, her guess will fail frequently. On the other hand, if she tries to eavesdrop, e.g., by interacting her qubits with those of Alice and/or Bob, this will necessarily induce errors between
the results of Alice and Bob that can easily be detected by them. Therefore she can easily be detected by checking the errors established between the results of Alice and Bob. Finally, using the well-established procedure, Alice and Bob can both arrive at the identical and secure random bit sequence that can be used as a key to encrypt information. For more details and various protocols, we refer the reader to the literature [2].

The first experimental realization [11] used polarization-encoded qubits with the simple identification $|0\rangle = |H\rangle$ and $|1\rangle = |V\rangle$, where $H$ and $V$ denote a horizontally and a vertically polarized photon, respectively (see Fig. 3.5).

**Fig. 3.6.** Scheme for entanglement swapping, i.e., the teleportation of entanglement. Two pairs of entangled qubits 0–1 and 2–3 are produced by two Einstein–Podolsky–Rosen (EPR) sources. One qubit from each of the pairs is sent to two separated observers, say qubit 0 is sent to Alice and qubit 3 to Bob. The other qubits 1 and 2 from each pair become entangled through a Bell-state measurement, whereby qubits 0 and 3 also become entangled. This requires the entangled qubits 0 and 3 neither to come from a common source nor to have interacted in the past.

Another protocol utilizing all three fundamental concepts, randomness, complementarity, and entanglement, is quantum teleportation [12,13], where one can transfer the quantum state of one system to another over arbitrary distances without physically transferring the system itself. The most interesting realization of quantum teleportation occurs when an entangled state itself is teleported, also called entanglement swapping [14,15]. In this experiment, as shown schematically in Fig. 3.6, one starts with two entangled pairs of qubits and performs a Bell-state measurement on one qubit from each pair. That way, the other two qubits, no matter how far they might be separated from each other, become entangled even though they share no common past. This protocol is conceptually very interesting, as it can be viewed as quantum teleportation of qubits which do not even have their own well-defined state. This is because an entangled qubit itself can only be described by a mixed density matrix. In a recent experiment [16], it has been possible to perform the Bell-state measurement with sufficient quality for the two outer, newly
entangled photons to become so highly entangled that the Bell inequality was violated. We might mention that such schemes may be of importance in future long-distance quantum communication protocols involving, for example, quantum repeaters [17].

There are many other applications of fundamental quantum concepts in new information technology protocols. These include, most notably, quantum computation, which is seen by many, including the present authors, as the future, albeit maybe long-term, of computation.

3.3 Quantum Physics as a Science of Information

The various debates about the conceptual significance of quantum mechanics can to a large extent be seen as a debate about what quantum physics refers to. Does it refer to reality directly or does it refer to (our) knowledge, and therefore to information? If quantum physics refers to reality, which reality is it? Is it the reality which appears to us, or is it a more complicated reality, like the one alluded to in the many-worlds interpretation?

We suggest that significant inspiration can be obtained from Niels Bohr, who, for example, according to Aage Petersen liked to say [18]: “There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature.”

To us, it is thus suggestive that knowledge is the central concept of quantum physics. In modern language, knowledge can be equated with information. Therefore, one needs first a proper measure of information. One might be tempted to use Shannon’s measure

\[ I = - \sum_i p_i \log p_i , \]  

(3.3)

where \( p_i \) is the probability of sign \( i \) occurring in a sequence. Yet it turns out that Shannon’s measure is not adequate to describe the knowledge gained in an individual quantum experiment [19]. This feature can be understood in various ways. The most central one is this: the fact that the Shannon measure of information contains the logarithm is related to the postulate that the information gained in a series of observations of different properties must be independent of the specific sequence in which the properties are read out. Clearly, such a requirement is no longer valid in quantum mechanics, unless the properties are commuting, which is in general an exception. So what is needed is a measure of information which accounts for complementarity and describes the total information obtainable in a complete set of quantum experiments. We have suggested elsewhere [20] that the most appropriate measure of information is

\[ I = \sum_i p_i p_i , \]  

(3.4)
which may be viewed as the sum of the individual probabilities weighed by these probabilities themselves. The total information content $I_{\text{total}}$ of a quantum system is then obtained as a sum of individual measures of information $I_j$ [of the type given in (3.4)] over a complete set of maximally mutually complementary observables (indexed by $j$)

$$I_{\text{total}} = \sum_j I_j = \sum_j \sum_i p_{ij} p_{ij}. \quad (3.5)$$

Here $p_{ij}$ denotes the probability of observing the $i$th outcome of the $j$th observable.

The notion of mutually complementary observables may need to be explained further. A famous example is given by the three spin components of a spin-1/2 particle taken along three directions orthogonal in space (not to be confused with orthogonal quantum states). From an operational point of view, two variables $A$ and $B$ are maximally mutually complementary if the knowledge of one completely precludes any knowledge of the other. In the case of spin, if $A$ represents the spin along the $x$-direction, then $B$ might represent the spin along any direction orthogonal to $x$ in space. It is a well known feature that if the spin along $x$ is well defined, the spin along these other directions is maximally undefined. To come back to our example, the sum $\sum_j$ in (3.5) in the case of a spin-1/2 particle has to be taken along any three spatially orthogonal directions, i.e., $j = x, y, z$.

It has not escaped our attention that (3.5) can be put onto a nicely visualizable foundation if one defines an information space spanned by mutually complementary observables [26]. Then $I_{\text{total}}$ just represents the square of the length of a vector in that information space when the square of the length of individual components is just given by $I_j$.

If, as we have suggested above, quantum physics is about information, then we have to ask ourselves what we mean by a quantum system. It is then imperative to avoid assigning any variant of naive classical objectivity to quantum states [21]. Rather it is then natural to assume that the quantum system is just the notion to which the probabilities in (3.4) and (3.5) refer, and no more. The notion of an independently existing reality thus becomes void.

We might therefore ask how much information a quantum system might carry, stressing again that by ‘carry’ we just refer to the total amount of information and not to the objective existence of any subject actually carrying the information.

It is obvious that a large system, being our mental representative of the information characterizing it, carries a lot of information, i.e., a great many bits. Then how does that amount of information scale with the size of the object? It is very suggestive to assume that the smaller a system, the less information it carries. One may even consider the amount of information carried by a system as defining its size. Basically, we postulate that:
1. the amount of information carried by any system is finite,
2. the amount of information is lesser the smaller the system.

These assumptions may be supported by referring to Feynman [22]: “It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of space/time is going to do?” Evidently, Feynman’s problem is solved if the ‘tiny piece of spacetime’ only contains a finite amount of information, and the less, the smaller the piece is.

We arrive at a natural limit when a system only represents one bit of information. Once that is achieved, the system can only represent the yes/no answer to one question. If the system is asked another question, the answer by necessity has to be random. Thus, randomness is a fundamental feature of our world [23, 24]. This, we suggest, also provides a natural foundation for complementarity. Consider, for example, a simple two-path interferometer as shown in Fig. 3.7.

![Two-path interferometer diagram]

Fig. 3.7. Two-path interferometer. The source emits coherent waves of which two beams are selected and incident on the beam-splitter. Each of the two beams has the same amplitude for being transmitted or reflected at the beam-splitter, and the outgoing beams are thus coherent superpositions of the incoming ones.

As is well known, in such an interferometer, we can prepare the state such that either the path $|a\rangle$ or the path $|b\rangle$ is taken by a particle. In that case, the trajectory after the semi-reflecting beam-splitter is completely random, or, in other words, detectors I or II will each register the particle with the same probability of 50%. On the other hand, we can prepare the state in a coherent
leads to conceptually difficult situations. The conceptual problem is in our
view related to the fact that we wish to define all notions on operationally
verifiable bases or foundations, that is, on foundations which can be verified
directly in experiment. It is obvious that an infinite number of complemen-
tary observables can never be realized in experiment. In our opinion, it is
therefore suggestive that the concept of an infinite number of complementary
observables and therefore, indirectly, the assumption of continuous variables,
are just mathematical constructions which might not have a place in a final
formulation of quantum mechanics.

This leads to the second question, namely, how to derive the Schrödinger
equation. If the assumption just expressed is correct, namely that continuous
variables are devoid of operational and therefore physical meaning in quan-
tum mechanics, there is no need to express the Schrödinger equation based
on continuous variables in our new language. Indeed, one should then refer to
situations where one always has only a finite number of complementary ob-
servables. In our opinion such a point of view is experimentally well founded,
as any experiment will always lead to only a finite number of bits and a finite
number of the experimental results on the basis of which only a finite number
of observables can be operationally defined.

It has not escaped our attention that our way of reasoning also leads to
new possibilities for understanding why we have quantum physics, i.e., for
answering Wheeler’s famous question: Why the quantum? Identifying sys-
tems with the information they carry, we note that information is necessarily
quantized. One can have one proposition, two propositions, three proposi-
tions, etc., but obviously the concept of, say, $\sqrt{2}$ propositions is devoid of any
meaning. Therefore, since information is quantized that way, our description
of information, which is quantum mechanics, also has to be quantized.

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