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Quantum information processing with cold atoms and trapped ions

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Abstract
This paper summarizes some important achievements of quantum information processing with trapped ions or neutral atoms. In particular, we describe the storage of information and realization of two-qubit gates with ions, as well as the creation of entanglement and quantum simulation with cold atoms in optical lattices.

1. Introduction

In this anniversary, we should recall two important contributions of Einstein to quantum mechanics. On the one hand, we have the photoelectric effect, which established the quantum nature of light. On the other hand, we have the well-intentioned paper by Einstein, Podolski and Rosen (Einstein *et al* 1935), in which the authors questioned the validity of quantum mechanics by attacking the existence of entangled states as real entities. While the first work belongs to the foundations of quantum optics, the second paper resulted in an interesting spinoff, which, through the papers of Bell proving that quantum mechanics allows stronger correlations than classical theories (Bell 1965, 1966), resulted in the birth of quantum information theory.

In the last few years we have seen an astonishing success in atomic, molecular and optical physics (AMO) in perfectly controlling the quantum states of atoms and photons, and suppressing decoherence due to unwanted interactions with the environment. These days we can not only disprove Einstein's arguments about the incompleteness of quantum mechanics by studying travelling photon pairs, but we can engineer interesting quantum states, and create and manipulate entanglement as a resource, as suggested by quantum information theory. Furthermore, once the control of small systems (ions, atoms, photons) is mastered, the next milestone in the coming years will be to put together many of these basic blocks, building more complex systems with applications in quantum information processing or precision measurements (Nielsen and Chuang 2002)[4].

Two particular systems are the main characters in the success of AMO physics and will be discussed in this paper: they are laser cooled trapped ions (Cirac and Zoller 1995, Kielpinski *et al* 2002, Cirac and Zoler 2000, Monroe *et al* 1995, Schmidt-Kaler *et al* 2003, Leibfried *et al* 2003) and cold atoms in optical lattices (Jaksch *et al* 1998, 1999, Greiner *et al* 2002, Mandel *et al* 2003, Porto *et al* 2003). If we were to remark on an important difference between both systems we would mention the size. Experiments with trapped ions have been developed focusing on the control at the single ion level, being able to modify the quantum state of a particular ion in small traps and making any two ions interact. These setups have been used for the creation of entanglement, precision measurement and small-scale quantum computations, and are thus the most promising candidates for scalable quantum computing. Cold atoms, on the other hand, provide us with a huge number of subsystems, particles or qubits. The difficulty of addressing individual atoms is compensated for by the simplicity with which they simulate many interesting Hamiltonians, and the potential for performing quantum gates in a massive parallel way. This makes cold atoms best suited for applications in condensed matter physics via quantum simulation, and also in the creation of entangled states of many particles.

The outline of this paper is as follows. First we will briefly introduce some concepts of quantum information, such as the notion of quantum register and quantum computer, so as to motivate the following two sections about physical implementations. Second, we will talk about quantum computing with trapped ions. We will explain how the information is stored and manipulated in the ions, focusing on the first design by Cirac and Zoller (1995) and moving to issues such as fast quantum gates (García-Ripoll *et al* 2003) and scalability. An overview of recent experimental achievements will also be presented. Third, we will move on to cold atoms in optical lattices. The physics behind these systems will be briefly explained, together with current proposals and experiments for performing massive entanglement and quantum simulation in these setups. Finally we will talk about perspectives in these two fields.

2. Quantum information and computation

The basic element of quantum computing is the qubit, i.e. a two-level or spin-1/2 system $|0\rangle, |1\rangle$. A string of N qubits provides a quantum register. The general quantum state of the register is an entangled state

$$|\Psi\rangle = \sum_{\{x_i=0,1\}} c_{x_{N-1}x_{N-2}\ldots x_0}|x_{N-1}x_{N-2}\ldots x_0\rangle$$

in the 2^N-dimensional product Hilbert space of the qubits. A quantum computation corresponds to unitary operations on the state of the quantum register, $|\psi\rangle \longrightarrow \hat{U}|\psi\rangle$. These unitary operations $\hat{U}$ can be decomposed into a sequence of single- and two-qubit quantum gates. A single-qubit gate corresponds to the general rotation of the spin-1/2 representing the qubit, while a two-qubit gate is a nontrivial entanglement operation of a pair of qubits, for example in the form of a Controlled-NOT $|x_1\rangle|x_2\rangle \longrightarrow |x_1\rangle|x_1 \oplus x_2\rangle$ where $\oplus$ denotes addition modulo 2, or a phase gate $|x_1\rangle|x_2\rangle \longrightarrow (-1)^{x_1x_2}|x_1\rangle|x_2\rangle$. The final step is a read-out of the state of the qubits (see figure 1).

[4] For a general review on implementations see, for example, *Quantum Inf. Comp.* vol 1 (number 2 and 3); and Quantum Information Science and Technology Roadmap http://qist.lanl.gov.

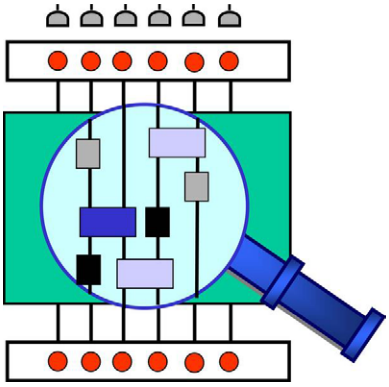

Figure 1. Quantum computer. Qubits are prepared in the initial state (lower circles), then processed using a universal set of quantum gates (centre), and their final state is detected (upper circles).

The physical requirements for implementing a quantum computer are summarized in the *DiVincenzo criteria* (see footnote 4). Roughly, one must be able to set the register to a well-known value, apply any unitary operation by combining the gates of a finite universal set, and finally measure the state of the qubits. A quantum computer setup should also be scalable to a large number of qubits, and the gates must operate with a very high fidelity in order to be able to apply error correcting codes (Shor 1995, Steane 1996).

2.1. Quantum simulation

An important feature of a quantum computer is that it is capable of simulating the dynamics of any other quantum mechanical system. Before getting into the details of how this works, let us briefly expose the complexity of performing these simulations numerically.

For simplicity, let us consider a chain of N spins or qubits or two-level systems, which is modelled by some Hamiltonian with nearest neighbour and maybe some local terms

$$H = \sum H_{\text{int}}^{k,k+1} + H_{\text{loc}}^{k}.$$

We want to study the evolution of a quantum state, that is we want to compute $|\psi(t)\rangle = \exp(-\mathrm{i}Ht)|\psi(0)\rangle$. Due to the form of the Hamiltonian, we can first decompose the unitary as a product of short-time evolution operators, $\exp(-\mathrm{i}Ht) = [\exp(-\mathrm{i}H\Delta t)]^M = U^M$, which act for a time $\Delta t = t/M \ll t$, and then into a product of single- and two-particle operations:

$$U \simeq \mathrm{e}^{-\mathrm{i}(H_{\text{int}}^{1,2}+H_{\text{loc}}^{1})\Delta t}\mathrm{e}^{-\mathrm{i}(H_{\text{int}}^{2,3}+H_{\text{loc}}^{2})\Delta t}\cdots\mathrm{e}^{-\mathrm{i}(H_{\text{int}}^{N,1}+H_{\text{loc}}^{N})\Delta t}. \qquad (1)$$

A way to solve this problem is to represent the state as a vector of 2^N complex numbers in some basis of the Hilbert space and perform the operations in (1) numerically. These are just two- and single-qubit operations and can be done efficiently. However, as soon as we reach a number of particles of the order of 30, we will not be able to store the vector in any existing computer and the time required to simulate the action of the gates will increase exponentially with the number of qubits.

This is not the case for quantum computers. With a quantum register we only need N qubits to represent the state of N particles or spins, and the unitary operations in (1) may be performed efficiently in parallel in a time proportional to the number of gates and thus to N (see figure 2). Therefore, we see that with a quantum computer it is much more efficient to

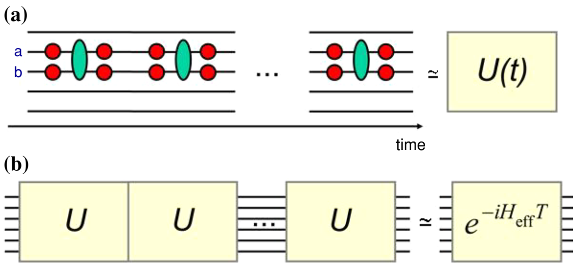

Figure 2. Quantum simulator: (a) time evolution for a single time step $U(t)$ is built up from a sequence of single and two-qubit gates (red and green circles) operating on qubits (denoted by a and b). (b) The effective time evolution operator $\exp -\mathrm{i}H_{\mathrm{eff}}t$ is built from a sequence of operators $U(t)$.

simulate quantum systems, something that Feynman already pointed out in 1982 (Feynmann 1982, 1986).

Another possibility, potentially simpler than that described above and closer to the spirit of Feynman's proposal, is to use an 'analogue' quantum computer to do the job. Instead of building a physical system that is able to produce all two- and single-qubit operations that we wish, we choose a physical system that can be very well controlled and measured, and which is described by the same Hamiltonian that one wants to simulate. As we will see below, this is the case of cold atoms in optical lattices.

While there are many fields of physics that would benefit from the existence of quantum simulators, maybe the most prominent one is condensed matter physics. There are many open questions related to important models (Hubbard, spin Hamiltonians, etc), which cannot be attacked with current numerical machinery, and which, as we will show later, have a potentially simple solution using cold atoms or even trapped ions.

3. Cold trapped ions

Right after the discovery of Shor's factoring algorithm in 1994 (Shor 1997), trapped ions interacting with laser light were suggested as one promising candidate to build a small-scale quantum computer (Cirac and Zoller 1995). The reason is that, for many years, the technology to control and manipulate single (or few) ions had been very strongly developed in the fields of ultrahigh precision spectroscopy and atomic clocks (Leibfried *et al* 2003). In particular, ions can be trapped and cooled such that they remain practically frozen in a specific region of space; their internal states can be precisely manipulated using lasers, and one can perform measurements with practically 100% efficiency; they also interact with each other very strongly due to Coulomb repulsion, and they can, at the same time, be decoupled from the environment very efficiently.

3.1. Ions as quantum computers

The ions, once stored and laser cooled in an electromagnetic trap (see figure 3), can be described using a finite number of external and internal degrees of freedom. The first ones are the collective vibrational modes associated with the motion of the ions around their equilibrium positions. Note the word 'collective'; that emphasizes the fact that if one of these phonon modes is excited, all ions participate in the motion. The internal degrees of freedom, on the other hand, involve the electronic structure, and the electronic and nuclear spin of each ion.

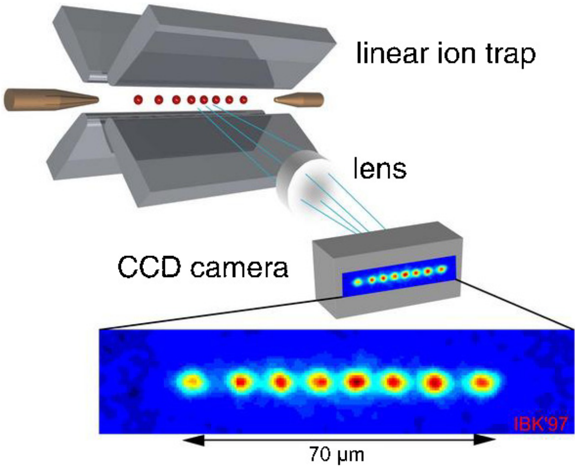

Figure 3. String of ions in a linear trap (upper part) and a CCD image (lower part) (courtesy of R Blatt at the University of Innsbruck).

Of an infinite set of discrete atomic levels, two states, which are denoted by $|0\rangle$ and $|1\rangle$, are used to store a qubit. Such states, which are ground hyperfine or metastable Zeeman levels, have very long lifetimes, sometimes rounding the years. This feature prevents the quantum information from being distorted during the quantum gates.

To start the computation, one can prepare all the qubits in state $|0\rangle$ by using optical pumping techniques, in which whenever the ion is in a state different to $|0\rangle$ it absorbs a photon and decays into another state until it finally reaches the desired state. At the end of the computation, one can read out the state of the ions by performing measurements based on the quantum jump technique (Bergquist *et al* 1986, Nagourney *et al* 1986, Sauter *et al* 1986). The idea is to illuminate the ions with laser light of appropriate frequency and polarization so that if an ion is in the state $|0\rangle$ it does not absorb photons, whereas if it is in $|1\rangle$ it absorbs and emits photons. Whenever fluorescence is detected or not, this indicates that the atom has been measured in states $|1\rangle$ and $|0\rangle$, respectively.

The computation itself requires the implementation of single- and two-qubit gates. The first ones can be carried out on each atom independently by coupling their internal states $|0\rangle$ and $|1\rangle$ with a laser (or two, in Raman configuration). By adjusting the frequency and intensity of the laser, one can carry out arbitrary single-qubit gates.

For the two-qubit gates, on the other hand, we need to make pairs of qubits interact in a controlled way. In the case of trapped ions, the interaction is mediated by the vibrational modes of the trap (Cirac and Zoller 1995), which, as we mentioned before, are collective and thus sensed by all ions. The key idea introduced by Cirac and Zoller (1995) was a means to make the state of a vibrational mode depend on the internal state of a given ion. By illuminating that one ion with laser light tuned to the appropriate sideband (Cirac *et al* 2002), it is possible to map the quantum information, $|0\rangle$ or $|1\rangle$, to a state with 0 or 1 phonons on the centre of mass mode, for instance[5]. Combining this operation with a similar one on a second ion, it is

[5] The fact that the laser couples the internal and external degrees of freedom of the ions is a simple consequence of the fact that each time an ion absorbs or emits a photon, not only the internal but also the motional state is changed due to the photon recoil.

possible to perform a gate

$$\begin{aligned} |\epsilon_1\rangle_i \otimes |\epsilon_2\rangle_j \otimes |\Psi\rangle &\to |\epsilon_1'\rangle_i \otimes |\epsilon_2'\rangle_j \otimes |\Psi'_{\epsilon_1,\epsilon_2}\rangle \\ &\to -(-1)^{\epsilon_1\epsilon_2}|\epsilon_1'\rangle_i \otimes |\epsilon_2'\rangle_j \otimes |\Psi'_{\epsilon_1,\epsilon_2}\rangle \\ &\to -(-1)^{\epsilon_1\epsilon_2}|\epsilon_1\rangle_i \otimes |\epsilon_2\rangle_j \otimes |\Psi\rangle, \end{aligned}$$

which restores the vibrational degrees of freedom, $|\Psi\rangle$, and puts a phase that depends solely on the states of the ions, $\epsilon_{1,2} = 0, 1$.

3.2. Fast quantum gates and scalability

The specific way in which the two-qubit gate was implemented in the proposal (Cirac and Zoller 1995) required that the ions be at zero temperature and that they can be individually addressed by the laser beam without affecting the rest. In recent years, various clever ways of simplifying these requirements have been proposed by different groups (Molmer and Sorensen 1999, Cirac and Zoler 2000, Milburn *et al* 2000, Jonathan *et al* 2000, Leibfried *et al* 2003). However, here we will focus on the ideas from García-Ripoll *et al* (2003), where a new family of gates is developed that makes use of all vibrational degrees of freedom in the trap.

The idea behind these new gates is the following. When a harmonic oscillator is subject to external forcing, the orbits in the phase space of the oscillator are perturbed. In the language of coherent states, the instantaneous state is written as

$$|\psi(t)\rangle = \mathrm{e}^{\mathrm{i}\phi}|x(t) + \mathrm{i}p(t)\rangle = \mathrm{e}^{\mathrm{i}\phi}|\mathrm{e}^{-\mathrm{i}\omega t}[x(0) + \mathrm{i}p(0) + \delta z(t)]\rangle.$$

Here $x(t)$ and $p(t)$ are the position and the momentum of the oscillator in phase space. The motion is a composition of a rotation, due to the harmonic restoring force, plus a displacement, $\delta z(t)$ that is due to the external forces and can be cancelled. The important point is that there is an additional term, a phase, $\phi(t)$, that depends on the area covered by the oscillator on phase space. As was shown in Leibfried *et al* (2003) and in García-Ripoll *et al* (2003), this phase can be used to implement a quantum gate.

To turn the 'geometrical' phase into a gate, one has to apply state-dependent forces on the ions. This is done by illuminating the ions with a laser that induces an ac Stark shift only on one of the two atomic states, $|0\rangle$ or $|1\rangle$, or by sending fast laser pulses that change the internal state of the ion as well as the motional one (García-Ripoll *et al* 2003). These state-dependent forces make the trajectories of the ions in phase space depend on their internal state, so that the total phase (or area) acquired has the usual dependence $\phi = J\epsilon_1\epsilon_2$ needed to perform a phase gate (see figure 4).

A crucial point is to ensure that the displacements, δz, associated with the different vibrational modes in the trap are cancelled. If this condition is matched, the gate becomes insensitive to the actual vibrational state of the ions, and there is no need to laser cool the ions beforehand. Moreover, it has been shown in García-Ripoll *et al* (2003) that this gate can be performed in an almost arbitrary time, which can be three orders of magnitude smaller than the period of the trap that confines the ions, thus reaching the physical limits of ion trap quantum computing. The tolerance to temperature and the speed of the gate make it an ideal candidate for scalability, as we will see below.

3.3. Experimental achievements

The experimental verification of these ideas began in 1995 (Monroe *et al* 1995) with a proof of principle experiment in which a two-qubit quantum gate was realized. Since then, several

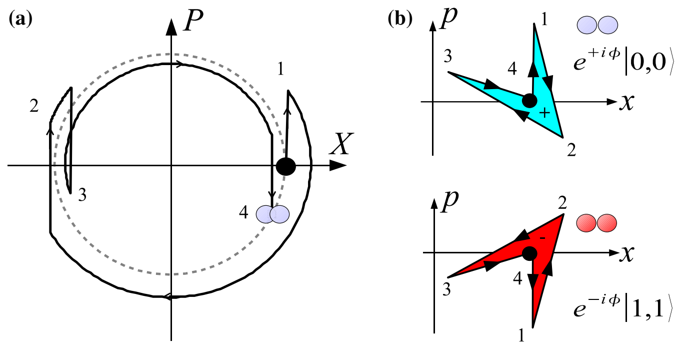

Figure 4. (a) Trajectory in phase space of the centre-of-mass state of the ions during a two-qubit gate (solid line). The time evolution consists of a sequence of instantaneous forces (vertical displacements), which are interspersed with free harmonic oscillator evolution (motion along the arcs). The appropriate forcing restores the vibrational mode on the original orbit of harmonic evolution (dashed circle). This particular plot corresponds to a four-pulse sequence given in García-Ripoll *et al* (2003). Figure (b) shows how the area and the phase depend on the internal state of the ions.

milestones have been achieved, especially in the laboratories led by David Wineland at NIST and Rainer Blatt in Innsbruck. Several versions of two-qubit gates have been carried out leading to very high efficiencies (Leibfried *et al* 2003, Schmidt-Kaler *et al* 2003), the so-called Deutsch–Jozsa algorithm (Gulde *et al* 2003) with a single ion has been implemented in Innsbruck, three- and four- particle entangled states have been prepared by performing small quantum computations in these labs (Roos *et al* 2004, Sackett *et al* 2000) and even small scale quantum teleportation experiments have been performed (Riebe *et al* 2004). On the other hand, the progress in manipulating ions has led to other interesting applications, such as precision measurement of Guthöhrlein *et al* (2001) and single-photon sources (Keller *et al* 2004, Maurer *et al* 2004).

The main obstacle to scale up the current setups is based on the fact that as the number of ions in the trap is increased, it becomes harder to only affect the desired ions with the laser, without affecting the rest, something which spoils the computation. About three years ago, new proposals to overcome this obstacle emerged (Kielpinski *et al* 2002, Cirac and Zoler 2000). The idea proposed in Kielpinski *et al* (2002) is to separate the region where the ions are stored from the one in which the gates take place (see figure 5). In order to perform a gate, the ion (or ions) are moved from the storage region to the gate region, something which does not disturb their internal state (since, as mentioned above, the Coulomb interaction is independent of that unless we couple them with a laser). There, they are driven by lasers to perform the gate, and then they are moved back to the storage region. The additional heating of the ions motion due to this transfer can be removed by cooling an ion of a different species which, on the one hand cools the other sympathetically and, on the other, does not disturb their internal states. Preliminary experiments demonstrating all the basic elements of this proposal have been successfully carried out at NIST. In view of these experiments, we see at present no fundamental obstacle to achieving scalable quantum computation in these systems.

In the near future, we expect a very crucial experimental progress with trapped ions. Very likely, proof of principle experiments demonstrating quantum error correction (Shor 1995, Steane 1996) and other intriguing properties of quantum mechanics will take place with three to six ions. When the technology allows us to reach 30 ions (e.g. using the scalable proposals) a new avenue of experiments will open up. In that case, one could start performing

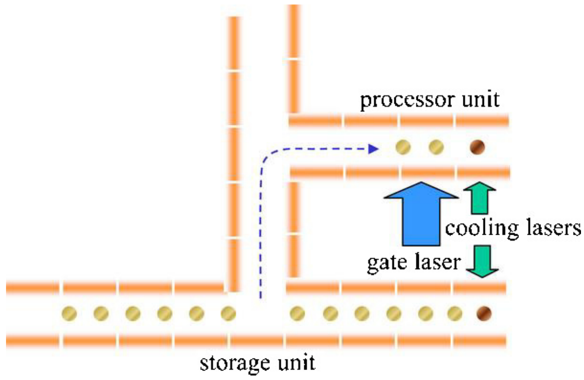

Figure 5. Scalable scheme for quantum computing with trapped ions as proposed in Kielpinski *et al* (2002). Internal states of ions serve as quantum memory. To perform one- or two-qubit gate operations ions are moved from the storage to a processing area. Heating due to transport can be suppressed by sympathetic cooling with laser-cooled ions (redrawn from Kielpinski *et al* (2002)).

computations which would compete with the most powerful classical computers that we have nowadays. Whether it will be possible to scale up the present setups up to several hundred thousands of ions or not, something which is required to factor 200 digit numbers using Shor's algorithm and which requires fault-tolerant error correction (Nielsen and Chuang 2002), is still an open question. What we know by now is that there is no fundamental obstacle to achieving this goal, but it only depends on the capability of developing the appropriate technologies. We would like to emphasize that trapped ions are the only system where this strong statement can be made nowadays.

4. Cold atoms in optical lattices

There is another physical system that can be used as a quantum register. As first proposed by Jaksch *et al* (1998, 1999), it is possible to load a big number of ultracold atoms from a Bose–Einstein condensate into an optical lattice. The lattice is nothing but a standing wave of light that confines the atoms either to the minima or to the maxima of intensity. In the limit of strong confinement, the system behaves as a Mott insulator, allowing us to have one atom per lattice node. Using the internal state of these atoms as qubits, it is possible to entangle them in a massively parallel operation with spin-dependent confinement (Jaksch *et al* 1999) (see also Brennen *et al* (1998)). Such possibilities have been realized in a remarkable series of experiments in Munich (Greiner *et al* 2002, Mandel *et al* 2003).

4.1. Mott insulator to superfluid transition

As mentioned before, optical lattices behave as arrays of microtraps for cold atoms. Due to the low temperatures, atoms loaded in an optical lattice will only occupy the lowest Bloch band and their physics will be modelled by a simple Hubbard Hamiltonian (Jaksch *et al* 1998)

$$H = -\sum_{\langle i,j\rangle,\sigma} J_{ij} b^{\dagger}_{i\sigma} b_{j\sigma} + \frac{1}{2}\sum_{i} U_{\sigma,\sigma'} b^{\dagger}_{i\sigma} b_{i\sigma} b^{\dagger}_{i\sigma'} b_{i\sigma'}. \qquad (2)$$

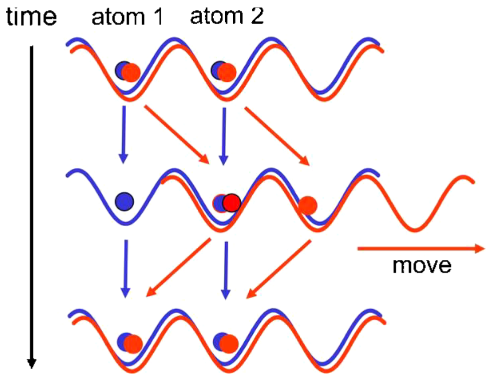

Figure 6. Controlled collisions of two atoms with internal states $|0\rangle$ and $|1\rangle$ (red and blue circles) in a movable state-dependent optical lattice (red and blue lattice) to entangle two atoms (Jaksch *et al* 1999, Mandel *et al* 2003). This scheme underlies the quantum simulator on the optical lattice.

Here $b_{i\sigma}$ and $b_{i\sigma}^{\dagger}$ are the destruction and creation operators for atoms at each lattice site, with given spin $\sigma = \pm\frac{1}{2}$; J_{ij} are hopping matrix elements connecting two lattice sites via tunnelling and U arises from the short-range interactions of atoms laying on the same site.

Remarkably, the ratio of kinetic energy versus interaction, J_{ij}/U, can be controlled in a time-dependent way by changing the intensity of the lasers that create the lattice. Weak confinement leads to large hopping terms, weak interactions and a superfluid phase. However, for strong confinement the hopping is exponentially suppressed and the atoms undergo a quantum phase transition to a Mott insulator state (Jaksch *et al* 1998). Under certain conditions this Mott insulator regime may have just one particle per site, providing a *very large* number—around 10^5—number of identifiable atoms whose internal states serve as qubits. While the quantum phase transition has been demonstrated in Munich (Greiner *et al* 2002), there remains the experimental challenge of creating perfectly loaded lattices, without holes.

4.2. Massive entanglement

The entanglement of these huge quantum registers is achieved by combining the collisional interactions with a *spin-dependent* optical lattice (Jaksch *et al* 1999). By a clever selection of the atomic states and laser configurations, it is possible to create two overlapping optical lattices which confine the atoms in states $|0\rangle$ and $|1\rangle$ independently. These lattices can be moved in order to produce state-dependent collisions between neighbouring atoms, as illustrated in figure 6. In this proposal only a quantum state with the first atom in $|1\rangle$ and the second atom in $|0\rangle$ will pick up a collisional phase ϕ. In a general setup with N ions this amounts to a quantum gate

$$U = \mathrm{e}^{\mathrm{i}\phi\sum_k |01\rangle_k\langle 01|} = \mathrm{e}^{\mathrm{i}\sum_k \frac{1}{4}\phi(1-\epsilon_k)(1+\epsilon_{k+1})}. \tag{3}$$

Up to some local operations that can be undone, $\mathrm{e}^{\pm\mathrm{i}\epsilon_i^z}$, the previous operation is a set of phase gates between neighbouring atoms. If the atoms are prepared initially in a superposition of $|0\rangle$ and $|1\rangle$, a single movement of the lattice will entangle in parallel all qubits. For two and three atoms this leads to the creation of Bell and GHZ states, respectively, and for larger setups, such as 2D lattices, this allows the generation of a cluster state (Briegel and Raussendorf 2001), which is the basic resource for universal quantum computing in Briegel *et al*'s one way quantum computer (Raussendorf and Briegel 2001).

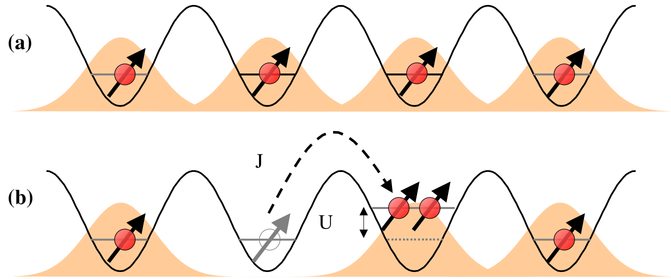

Figure 7. (a) A perfect Mott insulator of strongly interacting atoms ($J \ll U$) with internal states is equivalent to a set of spins. (b) When we allow some tunnelling between neighbouring sites, it is still unfavourable for two atoms to occupy the same site, but second order processes (here pictured as virtual hopping of atoms), lead to effective spin interactions, which are of order J^2/U (Duan *et al* 2003, García-Ripoll and Cirac 2003, Yip 2003, García-Ripoll *et al* 2004).

4.3. Quantum simulation

The parallelism inherent in the lattice movements makes 'atoms in optical lattices' an ideal candidate for a Feynman-type quantum simulator of bosonic, fermionic and spin many-body systems. Furthermore, the flexibility of the optical lattices allows for simulation of various types and strengths of particle interactions, changing geometries, in one, two or three dimensions and in a regime of many atoms, clearly inaccessible to any classical computer. By a stroboscopic switching of laser pulses and lattice movements combined with collisional interactions one can implement sequences of 1- and 2-qubit operations to simulate the time evolution operator of a many-body system (Jané *et al* 2003). For translationally invariant systems, there is no need to address individual lattice sites, which makes the requirements quite realistic in the light of the present experimental developments.

On the other hand, the controllability of these setups makes them also suitable for much simpler, 'analogic' quantum simulation. First of all, we get for free the possibility of studying bosonic and fermionic Hubbard Hamiltonians (2) with arbitrary interactions, doping rates and lattice geometries. This is itself of great interest, because the Hubbard model is believed to capture the physics behind hight-T_c superconductivity.

Furthermore, it is possible to stretch these settings to a regime of weak hopping and unit occupation in which the atoms behave as localized spins with tuneable interactions. Roughly, for large U/J having two atoms in the same site is energetically very expensive, and we can consider two sets of states: a ground state with unit occupancy and all possible spin states (figure 7(a)) and excited states with doubly occupied sites (figure 7(b)). By working with (2) up to second-order perturbation theory, one reaches an effective Hamiltonian

$$H_{\rm eff} = \sum_{\langle i,j\rangle} \left[\lambda^z_{i,j}\sigma^z_i\sigma^z_j + \lambda^{\perp}_{i,j}\left(\sigma^x_i\sigma^x_j + \sigma^y_i\sigma^y_j\right)\right],$$

where the couplings $\lambda \sim J^2/U$ are modified by changing the optical lattice. These ideas have been put forward in Duan *et al* (2003), García-Ripoll and Cirac (2003), Yip (2003), García-Ripoll *et al* (2004), and together with the flexible machinery for measuring correlation functions developed in García-Ripoll *et al* (2004) they show the potential utility of cold atoms for condensed matter physicists.

4.4. Perspectives

While most of the above discussion has focused on optical lattices, new designs of arrays of microtraps e.g. based on nano-optics or magnetic microtraps will be one way to allow individual addressing of atomic qubits and entanglement operations. The ideas reviewed above about how to implement quantum gates with atoms in optical lattices can easily be extended to these systems, and we expect that in the near future they will be implemented experimentally.

In the near future, we expect that atoms in optical lattices will be used to simulate a variety of other physical systems such as, for example, interacting fermions in two-dimensional lattices of various geometries. We also expect an important progress towards loading single (neutral) atoms in different types of potentials (optical, magnetic, etc), and the performance of quantum gates with few of these systems. This would allow us to create a few atom entangled states which may be used to observe violations of Bell inequalities, or to observe interesting phenomena such as teleportation or error correction. As opposed to the trapped ions, at the moment it is hard to predict if scalable quantum computation will be possible with neutral atoms in optical lattices with the present experimental set-ups. In any case, due to the high parallelism of these systems, we can clearly foresee that they will allow us to obtain a very deep insight into condensed matter physics via quantum simulations.

5. Conclusions

In the last decade, the world of AMO physics has experienced enormous progress in the trapping and quantum control of atoms, using lasers and electromagnetic fields. This will have immediate applications for quantum information processing, since now atomic systems are in a position to fulfil the requirements for building a quantum computer (see footnote 4).

In this review we have illustrated the previous statement with two physical systems: trapped ions and cold atoms in optical lattices. Both systems have different advantages and disadvantages, and while trapped ions seem to be closer to scalable quantum computing, cold atoms might produce the first real application of quantum information by simulating condensed matter physics models and producing highly entangled states of many particles.

There are other quantum optical systems that have experienced very remarkable progress during the last few years, and which may become equally important in the context of quantum information. Just to mention some examples, in the context of cavity QED groups at Caltech, Georgia Tech, Innsbruck, and Munich have trapped single atoms and ions inside cavities, and let them interact with the cavity field, which can be used as single (or entangled) photon(s) generators as well as to build quantum repeaters for quantum communication. Atoms have been trapped in several kinds of optical and magnetic traps, and they have been moved very precisely to different locations in space. Distant ensembles of atoms have been entangled using laser fields, and the quantum state of light has been recorded in the ensembles and read out.

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