

Probing topology by “heating”

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We reveal an intriguing manifestation of topology, which appears in the depletion rate of topological states of matter in response to an external drive. This phenomenon is presented by analyzing the response of a generic 2D Chern insulator subjected to a circular time-periodic perturbation: due to the system’s chiral nature, the depletion rate is shown to depend on the orientation of the circular shake. Most importantly, taking the difference between the rates obtained from two opposite orientations of the drive, and integrating over a proper drive-frequency range, provides a direct measure of the topological Chern number of the populated band (ν): this “differential integrated rate” is directly related to the strength of the driving field through the quantized coefficient $\eta_0 = \nu/\hbar^2$. Contrary to the integer quantum Hall effect, this quantized response is found to be non-linear with respect to the strength of the driving field and it explicitly involves inter-band transitions. We investigate the possibility of probing this phenomenon in ultracold gases and highlight the crucial role played by edge states in this effect. We extend our results to 3D lattices, establishing a link between depletion rates and the non-linear photogalvanic effect predicted for Weyl semimetals. The quantized effect revealed in this work designates depletion-rate measurements as a universal probe for topological order in quantum matter.

Introduction The quantization of physical observables plays a central role in our understanding and appreciation of Nature’s laws, as was already evidenced by the antique work of Pythagoras on harmonic series, and many centuries later, by the identification of the Balmer series in atomic physics [1]. More recently, in condensed-matter physics, the observation of quantized conductance unambiguously demonstrated the quantum nature of matter, in particular, the possibility for electronic currents to flow according to a finite set of conducting channels [2, 3]. While the quantized plateaus depicted by the conductance of mesoscopic channels depend on the samples geometry [3], a more universal behavior exists when a two-dimensional electron gas is immersed in an intense magnetic field [2]: In the non-interacting regime, the Hall conductivity is then quantized according to the Thouless-Kohmoto-Nightingale-Nijs (TKNN) formula [4], $\sigma_H = (e^2/h)\nu$, where h is Planck’s constant and where ν is a topological invariant – the Chern number – associated with the filled Bloch bands [5, 6]. Since the discovery of this integer quantum Hall (QH) effect, the intimate connection between topology and quantized responses has been widely explored in solid-state physics [7, 8], revealing remarkable effects such as the quantization of Faraday rotation in 3D topological insulators [9].

Building on their universal nature, topological properties are currently studied in an even broader context [10], ranging from ultracold atomic gases [11] and photonics [12, 13] to mechanical systems [14]. Interestingly, these complementary and versatile platforms offer the possibility of revealing unique topological properties, such as those emanating from engineered dissipation [15–17], time-periodic modulations [18–23], quantum walks [24, 25] and controllable interactions [11, 26, 27]. In ultracold gases, the equivalent of the TKNN formula was explored by visualizing the transverse displacement of an atomic cloud in response to an applied force [22]; the Chern number ν , and the underlying Berry curvature [8], were also extracted through state-tomography [28, 29], interferometry [30], and spin-polarisation measurements [31]. Besides, the propagation of robust chiral edge modes was identified in a variety of physical platforms [10, 11, 13, 14].

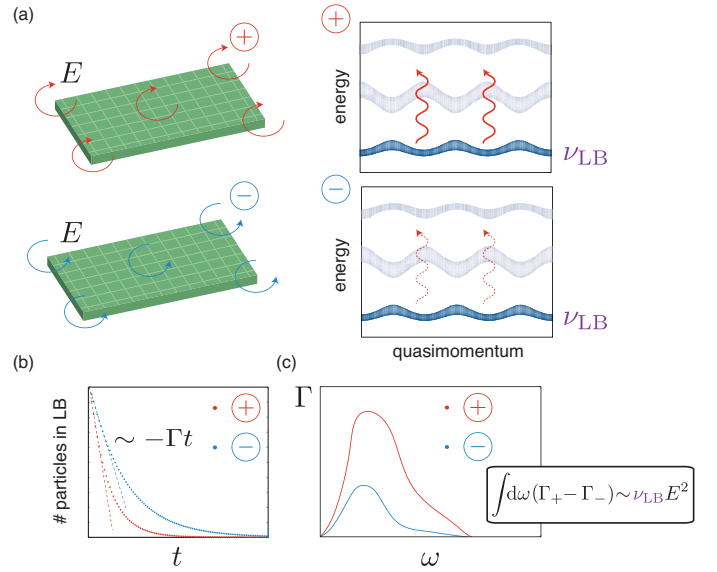


Figure 1. Topology through heating: (a) A 2D Fermi gas is initially prepared in the lowest Bloch band (LB) of a lattice, with Chern number ν_{LB} , and it is then subjected to a circular time-periodic modulation [Eq. (3)]. (b) The rate Γ associated with the depletion of the populated band, $\delta N_{LB}(t) \approx -\Gamma t$, is found to depend on the orientation of the drive, $\Gamma_+ \neq \Gamma_-$, whenever the LB is characterized by a non-trivial Chern number $\nu_{LB} \neq 0$. (c) Integrating the differential rate over a relevant drive-frequency range, $\Delta\Gamma^{\text{int}} = \int d\omega (\Gamma_+ - \Gamma_-)/2$, leads to a quantized result, $\Delta\Gamma^{\text{int}}/A_{\text{sys}} = (\nu_{LB}/\hbar^2)E^2$, where E is the strength of the drive and A_{sys} is the system’s area [Eq. (1)].

In this work, we demonstrate that the depletion rate of a Bloch band in a quantum lattice system, which reflects the non-linear inter-band response to a time-dependent perturbation, satisfies a quantization law imposed by topological properties. This observation of depletion-rate quantization suggests that *heating a system can be exploited to extract its topological order*. Specifically, our method builds on the chiral nature of systems featuring

Bloch bands with non-zero Chern number [7]. First, we find that the depletion rate of a circularly-shaken Chern insulator, as captured by Fermi's Golden Rule (FGR), crucially depends on the orientation (chirality) of the drive [Fig. 1]. Then, we identify an intriguing quantization law for the *differential integrated rate* (DIR), $\Delta\Gamma^{\text{int}}$, which is defined as the difference between the rates obtained from opposite orientations of the drive, integrated over a relevant frequency range. The quantization of the DIR can be simply expressed as

$$\Delta\Gamma^{\text{int}}/A_{\text{sys}} = \eta_0 E^2, \quad \eta_0 = (1/\hbar^2)\nu, \quad (1)$$

in terms of the drive amplitude E and the topological response coefficient η_0 ; here ν denotes the Chern number of the populated band and A_{sys} is the system's area. This result is in agreement with the intuition that the response of a trivial insulator ($\nu = 0$) to a circular drive should not depend on the latter's orientation. Interestingly, the quantized response (1) identified in this work is non-linear with respect to the strength of the driving field E , and it explicitly involves inter-band transitions, indicating that this phenomenon is distinct in essence from the TKNN paradigm [4], which is associated with linear transport and captured by single-band semiclassics [8, 32]. This quantized effect positions depletion-rate measurements as a versatile probe for topological order.

In the following, we demonstrate how the depletion rate associated with circularly-driven Chern insulators can be related to the topological Chern number ν , and we identify the relevant response coefficient in this context, $\eta_0 = \nu/\hbar^2$. We then discuss how this effect could be probed in realistic systems, setting the focus on how to avoid the detrimental contribution of edge-states in this framework. We then extend our results to 3D lattices, providing an instructive connection between topological depletion rates and the non-linear photogalvanic effect [33] recently predicted for Weyl semimetals [34].

Topology and quantization of depletion rates We start by studying a non-interacting (spinless) gas in a generic two-dimensional (2D) lattice, as described by the single-particle Hamiltonian \hat{H}_0 . In our study, we will assume that the lowest Bloch band (LB) of the spectrum, which is separated from higher bands by a bulk gap Δ_{gap} , is initially completely filled with fermions; the following discussion can be straightforwardly extended to other initial filling conditions. Considering systems with broken time-reversal symmetry, the topological properties of this LB will be accurately captured by the Chern number [5–7], henceforth denoted ν_{LB} . Thus, subjecting this system to a constant electric field $\mathbf{E} = E_y \mathbf{1}_y$ generates a total Hall current $\mathbf{J} = J^x \mathbf{1}_x$ satisfying the TKNN formula [4]:

$$J^x/A_{\text{sys}} = j^x = \sigma_{\text{H}} E_y, \quad \sigma_{\text{H}} = (q^2/h)\nu_{\text{LB}}, \quad (2)$$

where A_{sys} is the system's area and q is the charge of the carriers ($q = e$ in an electron gas). In gases of neutral atoms, such transport equations can be probed by measuring the flow of particles [32, 35–37] in response to a synthetic electric field (e.g. an optical gradient [22]); this latter situation corresponds to setting $q = 1$ in Eq. (2).

In this work, however, we are interested in the *depletion rate* of this system in response to a circular time-periodic perturbation, as described by the total time-dependent Hamiltonian

$$\hat{H}_{\pm}(t) = \hat{H}_0 + 2E [\cos(\omega t)\hat{x} \pm \sin(\omega t)\hat{y}], \quad (3)$$

where \pm refers to the two possible orientations (chirality) of the drive, (\hat{x}, \hat{y}) are the position operators [38], and ω is a frequency

used to drive inter-band transitions. Such circular shaking of 2D lattices can be implemented in cold atoms trapped in optical lattices [39], using piezo-electric actuators [21]; the Hamiltonian in Eq. (3) equally describes electronic systems subjected to circularly-polarized light [34, 40–42]. The total number of particles scattered and extracted from the LB, $N_{\pm}(\omega, t) \approx \Gamma_{\pm}(\omega)t$, is associated with the depletion rate Γ_{\pm} [Fig. 1], which can be accurately evaluated using Fermi's Golden Rule [43]

$$\Gamma_{\pm}(\omega) = \frac{2\pi}{\hbar} E^2 \sum_{e \notin \text{LB}} \sum_{g \in \text{LB}} |\langle e | \hat{x} \pm i\hat{y} | g \rangle|^2 \delta^{(t)}(\varepsilon_e - \varepsilon_g - \hbar\omega), \quad (4)$$

where $|g\rangle$ [resp. $|e\rangle$] denotes all the initially occupied [resp. unoccupied] single-particle states with energy ε_g [resp. ε_e], and where $\delta^{(t)}(\omega) = (2/\pi t) \sin^2(\omega t/2)/\omega^2 \rightarrow \delta(\omega)$ in the long-time limit [44]. The transitions to initially occupied states (g) are excluded in Eq. (4), as required by Fermi statistics, which is an important feature when many bands are initially occupied. We also point out that the number of scattered particles $N_{\pm}(\omega, t)$ can be directly detected in cold atoms by measuring the dynamical repopulation of the bands through band-mapping techniques [22].

For the sake of pedagogy, let us first analyze the excitation rate (4) in a frame where the total Hamiltonian (3) is translationally invariant. Performing the frame transformation generated by the operator

$$\hat{R}_{\pm} = \exp \left\{ i \frac{2E}{\hbar\omega} [\sin(\omega t)\hat{x} \mp \cos(\omega t)\hat{y}] \right\}, \quad (5)$$

the time-dependent Hamiltonian (3) is modified according to

$$\hat{H}_{\pm}(t) \approx \hat{H}_0(\mathbf{k}) + \frac{2E}{\hbar\omega} \left\{ \sin(\omega t) \frac{\partial \hat{H}_0(\mathbf{k})}{\partial k_x} \mp \cos(\omega t) \frac{\partial \hat{H}_0(\mathbf{k})}{\partial k_y} \right\}, \quad (6)$$

where we now adopted the momentum representation, and omitted higher-order terms in E in agreement with the perturbative approach [43] inherent to the FGR in Eq. (4) and below. In this frame, and taking the long-time limit, the excitation rate (4) now takes the more suggestive form

$$\Gamma_{\pm}(\omega) = \sum_{\mathbf{k}} \Gamma_{\pm}(\mathbf{k}; \omega), \quad (7)$$

$$\Gamma_{\pm}(\mathbf{k}; \omega) = \frac{2\pi E^2}{\hbar^3 \omega^2} \sum_{n>0} |\langle n | \frac{1}{i} \frac{\partial \hat{H}_0}{\partial k_x} \mp \frac{\partial \hat{H}_0}{\partial k_y} | 0 \rangle|^2 \delta(\varepsilon_n(\mathbf{k}) - \varepsilon_0(\mathbf{k}) - \hbar\omega).$$

Here, we introduced the initially-populated Bloch states of the LB, $|g\rangle \equiv |0(\mathbf{k})\rangle$, of dispersion $\varepsilon_0(\mathbf{k})$, as well as the initially-unoccupied Bloch states of the higher bands, $|e\rangle \equiv |n(\mathbf{k})\rangle$, of dispersion $\varepsilon_n(\mathbf{k})$ and band index n . Integrating the excitation rates $\Gamma_{\pm}(\omega)$ in Eq. (7) over all drive frequencies $\omega \geq \Delta_{\text{gap}}/\hbar$, i.e. activating all transitions between the filled LB and the higher bands [46], and considering the difference between these integrated rates, $\Delta\Gamma^{\text{int}} = (\Gamma_{\pm}^{\text{int}} - \Gamma_{\mp}^{\text{int}})/2$, defines the *differential integrated rate* (DIR), which reads

$$\Delta\Gamma^{\text{int}} = 4\pi(E/\hbar)^2 \text{Im} \sum_{n>0} \sum_{\mathbf{k}} \frac{\langle 0 | \partial_{k_x} \hat{H}_0 | n \rangle \langle n | \partial_{k_y} \hat{H}_0 | 0 \rangle}{(\varepsilon_0 - \varepsilon_n)^2}. \quad (8)$$

Comparing the latter with the expression for the Chern number [8]

$$\nu_{\text{LB}} = \frac{4\pi}{A_{\text{sys}}} \text{Im} \sum_{n>0} \sum_{\mathbf{k}} \frac{\langle 0 | \partial_{k_x} \hat{H}_0 | n \rangle \langle n | \partial_{k_y} \hat{H}_0 | 0 \rangle}{(\varepsilon_0 - \varepsilon_n)^2}, \quad (9)$$

we obtain the simple quantization law for the DIR per unit area in Eq. (1), with $\nu = \nu_{\text{LB}}$.

Remarkably, the integration inherent to the definition of the DIR reveals the Chern number of the ground band, while the properties of excited states drop out through the summation over all final states. The relation in Eq. (1) is reminiscent of the transport equation (2) associated with the QH effect: the DIR per unit area is directly related to the driving field E through a response coefficient η_0 that only depends on the topology of the populated band and on a universal constant (\hbar^{-2}). We point out that, contrary to the linear transport equation (2), the quantized response in Eq. (1) is *non-linear* with respect to the driving field, which highlights its distinct origin. In particular, the differential aspect of the measurement, which directly probes the chirality of the system by comparing its response to opposite shaking orientations, plays an essential role in this distinct quantized effect. Besides, we note that the latter explicitly involves inter-band transitions, ruling out the possibility of capturing it through a single-band semiclassical approach [8]. It is straightforward to generalize the result in Eq. (1) to situations where many bands are initially populated, in which case ν_{LB} should be replaced by the sum over the Chern numbers associated with these bands.

In the case of two-band models ($n = 1$), we point out that the *local* differential rate $\Delta\Gamma(\mathbf{k}; \omega) = [\Gamma_+(\mathbf{k}; \omega) - \Gamma_-(\mathbf{k}; \omega)]/2$ resulting from Eq. (7) is directly proportional to the Berry curvature $\Omega(\mathbf{k})$ of the LB [8]. Hence, measuring $\Delta\Gamma(\mathbf{k}; \omega)$ from wave packets (prepared in the LB and centered around \mathbf{k}) offers an elegant method to directly probe the geometrical properties of Bloch bands, as captured by the local Berry curvature [8]. Also, in that case, the allowed transitions are automatically restricted to $|0(\mathbf{k})\rangle \rightarrow |1(\mathbf{k})\rangle$, irrespective of Fermi statistics.

Practically, we propose that the integrated rates could be experimentally extracted from many individual measurements [22, 46], corresponding to sampled (fixed) values of ω : $\Gamma_{\pm}^{\text{int}} \approx \sum_l \Gamma_{\pm}(\omega_l) \Delta\omega$. This scheme could also be facilitated by the use of multi-frequency drives.

We have validated the quantization law in Eq. (1), based on a numerical study of the two-band Haldane model [64], in the topological phase where $\nu_{\text{LB}} = -1$. The rates $\Gamma_{\pm}(\omega)$ corresponding to a honeycomb lattice of size 100×100 , with periodic boundary conditions (PBC), are shown in Fig. 2. We verified that the DIR [Eq. (8)] obtained from this numerical data yields $\Delta\Gamma^{\text{int}}(\hbar^2/A_{\text{sys}}E^2) \approx -1.00$, in perfect agreement with the quantized prediction of Eq. (1).

On the effects of boundaries Importantly, the derivation leading to Eq. (1) implicitly assumed translational invariance and PBC (i.e. a torus geometry); in particular, this result disregards the effects related to the presence of (chiral) edge states in finite lattices [7]. Here, we reveal the important contribution of edge states, when considering more realistic systems with boundaries.

In order to analyze lattices with edges (and more generally, systems that do not present translational symmetry, such as disordered systems [47] or quasicrystals [48]), it is instructive to expand the modulus squared in the “real-space” formula (4) and then to integrate the latter over all frequencies ω ; this yields the integrated rates

$$\Gamma_{\pm}^{\text{int}} = (2\pi/\hbar^2)E^2 \sum_{g \in \text{LB}} \langle g | \hat{P}(\hat{x} \mp i\hat{y}) \hat{Q}(\hat{x} \pm i\hat{y}) \hat{P} | g \rangle, \quad (10)$$

where we introduced the projector $\hat{P} = 1 - \hat{Q}$ onto the LB. Then,

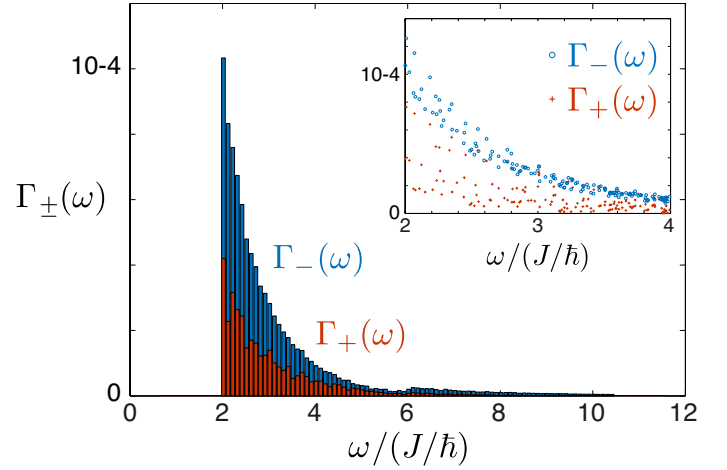


Figure 2. Depletion rates $\Gamma_{\pm}(\omega)$, as given by Eq. (7), for the driven two-band Haldane model with 10^4 lattice sites and PBC. The model parameters are set such that $\Delta_{\text{gap}} \approx 2J$, where J is the nearest-neighbor hopping amplitude; the strength of the drive [Eq. (3)] is $E = 0.001J/d$, where d is the lattice spacing. The inset shows the rates associated with all possible transitions, $\omega = (\varepsilon_e - \varepsilon_g)/\hbar$, while the main plot shows the averaged values, $\langle \Gamma(\omega) \rangle$, defined within each interval of width $\Delta\omega = 0.1J/\hbar$. Note the small peak around $\omega \approx 6J/\hbar$, which is associated with van Hove singularities in the density of states. The DIR [Eq. (8)] obtained from this data yields $\Delta\Gamma^{\text{int}}(\hbar^2/A_{\text{sys}}E^2) \approx -1.00$, in agreement with Eq. (1) and the theoretical prediction $\nu_{\text{LB}} = -1$. The rates $\Gamma_{\pm}(\omega)$ are expressed in units of J/\hbar .

the expression for the DIR [Eq. (8)] now takes the form

$$\Delta\Gamma^{\text{int}} = (\Gamma_+^{\text{int}} - \Gamma_-^{\text{int}})/2 = (E/\hbar)^2 \text{Tr} \hat{\mathcal{C}}, \quad (11)$$

$$\hat{\mathcal{C}} = 4\pi \text{Im} \hat{P} \hat{x} \hat{Q} \hat{y} \hat{P},$$

where $\text{Tr}(\cdot)$ is the trace. Importantly, when applying PBC, the quantity $(1/A_{\text{sys}})\text{Tr} \hat{\mathcal{C}} = \nu_{\text{LB}}$ is equal to the Chern number of the populated band [47], such that the result in Eq. (1) is indeed recovered in this real-space picture; in particular, this demonstrates the applicability of Eq. (1) to systems without translational symmetry and finite observation times.

The real-space approach allows for the identification of the strong edge-states contribution to the DIR $\Delta\Gamma^{\text{int}}$, when (realistic) open boundary conditions (OBC) are considered. To see this, let us recall that the trace in Eq. (11) can be performed using the position (or Wannier-state) basis $\{|r_j\rangle\}$; in particular, inspired by Refs. [47, 48], we decompose the DIR (11) in terms of bulk and edge contributions:

$$\Delta\Gamma_{\text{OBC}}^{\text{int}} = (E/\hbar)^2 \left\{ \sum_{r_j \in \text{bulk}} C(r_j) + \sum_{r_j \in \text{edge}} C(r_j) \right\}, \quad (12)$$

where we introduced the local marker $C(r_j) = \langle r_j | \hat{\mathcal{C}} | r_j \rangle$. As illustrated in Fig. 3, the local marker $C(r_j) \approx \nu_{\text{LB}}$ is almost perfectly uniform within the bulk of the system; in the thermodynamic limit, the bulk contribution $(\sum_{r_j \in \text{bulk}} C(r_j) \rightarrow A_{\text{sys}} \nu_{\text{LB}})$ leads to the quantized DIR predicted by Eq. (1) for PBC. However, the distinct contribution of the edge states, which is clearly identified at the boundaries in Fig. 3, is found to exactly compensate the bulk contribution. Consequently, the total DIR in Eq. (12) vanishes for OBC, $\Delta\Gamma_{\text{OBC}}^{\text{int}} = 0$, which is in agreement with the triviality of the underlying fibre bundle (the corresponding base space being flat [49]). This important observation shows

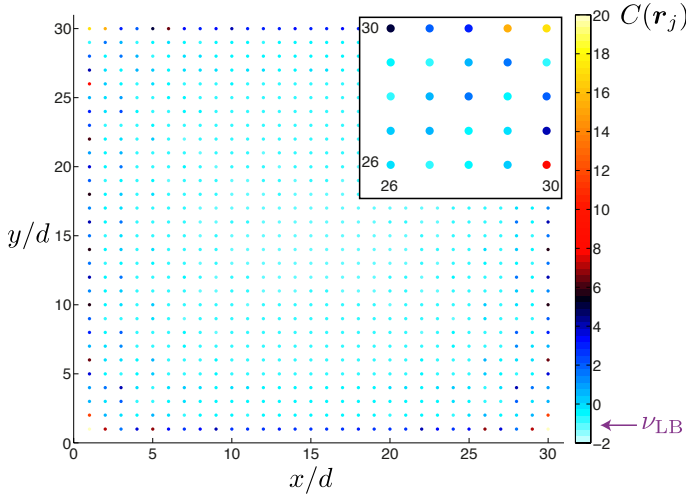


Figure 3. Local Chern marker $C(\mathbf{r}_j)$ in a 2D lattice with boundaries (OBC) realizing the Haldane model; d is the lattice spacing. Far from the boundaries, the marker is $C(\mathbf{r}_j) \approx -1$, in agreement with the Chern number of the populated band $\nu_{\text{LB}} = -1$. Close to the edges, the local marker is very large and positive (see the zoom shown in the inset) such that the total contribution of the edges exactly cancels the bulk contribution, $\sum_{\mathbf{r}_j} C(\mathbf{r}_j) = 0$: the DIR in Eq. (12) vanishes in a system with boundaries.

the drastic role played by the boundary in the present context; in particular, it indicates that the edge-states contribution must be annihilated in order to observe the quantized DIR [Eq. (1)] in experiments, as we further investigate in the paragraphs below.

Before doing so, let us emphasize that the detrimental contribution of the edge states cannot be simply avoided by performing a local measurement in the bulk, far from the edges. Indeed, probing the DIR in some region R would formally correspond to evaluating the quantity $\mu = (1/A_{\text{sys}})\text{Tr } \tilde{\mathcal{C}}$, where $\tilde{\mathcal{C}} = 4\pi\text{Im}(\hat{P}\hat{R}\hat{x}\hat{Q}\hat{R}\hat{y}\hat{P})$, and where \hat{R} projects onto the region R . While the local Chern number [47], defined as $\nu_R = (4\pi/A_{\text{sys}})\text{Tr Im}(\hat{R}\hat{P}\hat{x}\hat{Q}\hat{y}\hat{P}) \approx \nu_{\text{LB}}$, can indeed provide an approximate value for the Chern number of the LB, we find that μ strongly differs from the local marker ν_R , since $[\hat{R}, \hat{P}] \neq 0$.

Annihilating the edge-states contribution We now introduce two protocols allowing for the annihilation of the undesired edge-states contribution.

The first scheme consists in measuring the rate associated with the dynamical repopulation of the initially-unoccupied bulk bands only, i.e. disregarding the population of edge states. In practice, this requires the knowledge of the bulk-band structure. Formally, the resulting DIR would probe the quantity $\text{Tr } \hat{\mathcal{C}}$ in Eq. (11), but with the modified projector operator $\hat{Q} \rightarrow \hat{Q}_{\text{bulk}}$ that excludes the edge states of the spectrum. We have estimated the validity of this approach through a numerical study of the Haldane model with OBC, and found that the topological marker $\nu_{\text{bulk}} = (1/A_{\text{sys}})\text{Tr } \hat{\mathcal{C}}_{\text{bulk}}$ resulting from the modification $\hat{Q} \rightarrow \hat{Q}_{\text{bulk}}$ yields the approximate value $\nu_{\text{bulk}} \approx -0.85$ for a lattice with 2500 sites and $\nu_{\text{bulk}} \approx -0.91$ for a lattice with 10^4 sites; these results, which are close to the ideal value $\nu_{\text{LB}} = -1$, are found to be stable with respect to the Fermi energy (i.e. to the number of initially populated edge states) and improve as the system size increases. We then validated this scheme through a complete time-evolution of the circularly-driven Haldane model: we found that the resulting response coefficient in

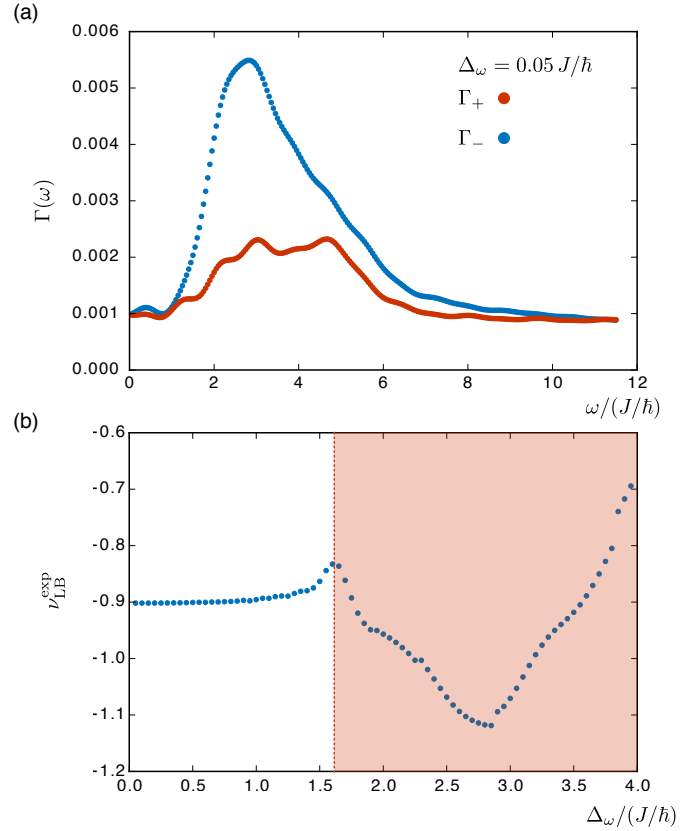


Figure 4. (a) Depletion rates $\Gamma_{\pm}(\omega)$ extracted from a numerical simulation of the circularly-shaken Haldane model with OBC. The edge-state contribution has been annihilated by initially confining the cloud in a disc of radius $r = 20d$, and then releasing it in a larger lattice of size 120×120 , after which the heating protocol (circular drive) was applied; other system parameters are the same as in Fig. 2. The rates $\Gamma_{\pm}(\omega_l)$, which are expressed in units of J/\hbar , were obtained by measuring the number of excited particles after a time $t = 4\hbar/J$, for fixed values of ω_l separated by $\Delta\omega = 0.05J/\hbar$. (b) Approximate value for the Chern number of the populated band $\nu_{\text{LB}}^{\text{exp}}$, as extracted from the rates through Eq. (1), and represented as a function of the step $\Delta\omega$ used to sample the drive frequencies; note that the area A_{sys} entering Eq. (1) corresponds to the initial area of the cloud. A satisfactory measure is reached when this sampling accurately probes the resonant peaks; we find $\Delta\omega \lesssim 0.5J/\hbar$ (i.e. at least 20 different frequencies) for an observation time $t = 4\hbar/J$. The saturation value $\nu_{\text{LB}}^{\text{exp}} \approx -0.9$ is limited by the fraction of particles populating the upper band, after abruptly removing the confinement, and can be improved by further increasing the initial radius r (or by softening the trap release).

Eq. (1) verified $\eta_0 \approx \nu_{\text{bulk}}/\hbar^2$, as estimated from the modified topological marker introduced above. This indicates how restricting the measurement of the depletion rate to the repopulation of bulk states only allows for a satisfactory evaluation of the quantized DIR in Eq. (1) under realistic conditions.

We then explore a more powerful scheme, which does not rely on the knowledge of the bulk band structure. Inspired by Ref. [65], we propose to initially prepare the system in the presence of a tight confining trap, and then to release the latter before performing the heating protocol. In this case, edge states associated with the full (unconfined) lattice remain unpopulated, as they do not couple to the time-evolving cloud upon the drive. We have validated this scheme through a complete time-evolution of the circularly-driven Haldane model, and we summarize the re-

sults in Fig. 4. Figure 4(a) shows the depletion rates $\Gamma_{\pm}(\omega_l)$, where ω_l are the many sampled frequencies; here, features of the sinc-squared function are visible due to the finite observation time [Eq. (4)]. Figure 4(b) shows the value of the extracted Chern number $\nu_{\text{LB}}^{\text{exp}}$, as a function of the frequency sampling step $\Delta\omega$; these values were obtained by comparing Eq. (1) to the numerical DIR, $\Delta\Gamma^{\text{int}} = \sum_l [\Gamma_+(\omega_l) - \Gamma_-(\omega_l)] \Delta\omega/2$. In this protocol, a residual deviation from the ideal DIR quantization is still visible, even in the limit $\Delta\omega \rightarrow 0$; see the saturation value $\nu_{\text{LB}}^{\text{exp}} \approx -0.9$ in Fig. 4(b). This is mainly due to the finite population of the higher band upon abruptly releasing the trap; we note that this weak effect is more pronounced for systems for which the Berry curvature is peaked around the gap-closing points (e.g. the Haldane model), and that it can be reduced by either increasing the initial size of the cloud or by softening the release of the trap. This numerical study demonstrates the validity and robustness of this scheme under reasonable experimental conditions, i.e. an observation time of a few hopping period and a limited number of sampled frequencies ω_l .

Beyond 2D We now illustrate how differential depletion rates associated with circular drive can probe topological matter in higher dimensions. We discuss two generic but distinct effects, which we concretely illustrate with Weyl-semimetal Hamiltonians [50–54].

The first effect stems from a direct generalization of the 2D analysis; it relies on noting that the expression for $\Delta\Gamma^{\text{int}}$ given in Eq. (8) does not depend on dimensionality, provided we consider spatial dimensions $D > 1$, where a chiral time-modulation is indeed well defined. In general, however, the sum over \mathbf{k} in Eq. (8) involves a D -dimensional FBZ, which leads to a non-quantized result. To see this, we generalize the drive operator in Eq. (3) as $\hat{V}_{\pm}(t) = E(\hat{a} \pm i\hat{b})e^{i\omega t} + \text{h.c.}$, where (\hat{a}, \hat{b}) are position operators defining the polarization plane. Using these notations, and considering the 3D case ($D = 3$), the DIR in Eq. (8) can be written as

$$\Delta\Gamma^{\text{int}}/V_{\text{sys}} = \eta_{3\text{D}}E^2, \quad \eta_{3\text{D}} = \mathbf{K} \cdot (\mathbf{1}_a \times \mathbf{1}_b)/2\pi\hbar^2, \quad (13)$$

where V_{sys} is the volume and \mathbf{K} is a vector with units of momentum. The response coefficient $\eta_{3\text{D}}$ is directly analogous to the general expression for the Hall conductivity in 3D, $\sigma^{ab} = (e^2/h)\epsilon^{abc}K_c/2\pi$, where \mathbf{K} is known to contain information on the topology of the bands [55, 56]. For instance, in the simplest case of a stacking of 2D Chern insulators, piled up along z with inter-plane separation d_z , and which is driven by a circular drive polarized in the x - y plane, we find the DIR in Eq. (13) with $K_z = 2\pi\nu_{\text{LB}}/d_z$, where ν_{LB} is the Chern number associated with each plane. In the context of time-reversal-breaking Weyl semimetals, a similar calculation identifies $\mathbf{K} = \nu_{\text{tot}}\mathbf{d}_W$, where \mathbf{d}_W is a vector connecting the Weyl nodes in k -space and ν_{tot} is the total Chern number of the occupied bands between these nodes [57–61]. We recall that such Weyl semi-metals can become 3D-QH insulators whenever the Weyl nodes meet at the edge of the FBZ, in which case the DIR in Eq. (13) is characterized by $\mathbf{K} = \nu_{\text{tot}}\mathbf{G}^0$, where \mathbf{G}^0 is a primitive reciprocal lattice vector [56, 62]. These results indicate that, similarly to the Hall conductivity in 3D, the DIR in Eq. (13) can probe topological properties of the bands, as well as non-universal properties (e.g. the Weyl node separation).

The above considerations require a protocol involving an integration over frequencies [Eq. (8)]. Our second protocol only involves a single frequency ω , and ultimately leads to a quantized signature stemming from the FGR in 3D lattices. It is based on a two-band analysis and builds on the observation that the dif-

ferential current $\Delta j^a = (j_+^a - j_-^a)/2$ is directly related to the *local* differential rate $\Delta\Gamma(\mathbf{k}; \omega) = [\Gamma_+(\mathbf{k}; \omega) - \Gamma_-(\mathbf{k}; \omega)]/2$, see Eq. (7), through

$$\frac{d\Delta j^a}{dt} = \int_{\text{FBZ}} \frac{d^3k}{(2\pi)^3} (v_1^a - v_0^a) \Delta\Gamma(\mathbf{k}; \omega), \quad (14)$$

where $v_{0,1}^a = \partial_{k_a}[\varepsilon_{0,1}(\mathbf{k})]$ are the two band velocities, and where we set the charge $q = 1$. In this protocol, we take the polarization plane of the drive to be perpendicular to the direction a . Then, noting that the δ -function in $\Delta\Gamma(\mathbf{k}; \omega)$ [Eq. (7)] defines a surface in k -space S orthogonal to the gradient $\partial_{k_a}(\varepsilon_1 - \varepsilon_0)$, leads to

$$\sum_{a=x,y,z} \frac{d\Delta j^a}{dt} = \frac{E^2}{8\pi^2\hbar^2} \int_S d\mathbf{S} \cdot \mathbf{\Omega} = \frac{E^2}{4\pi\hbar^2} \sum_i C_i, \quad (15)$$

where $\mathbf{\Omega}$ denotes the Berry-curvature vector [8] of the LB, and where the last sum extends over all momentum space monopoles i enclosed by the surface S with integer charge C_i . If all monopoles in the FBZ lie inside S then $\sum_i C_i = 0$. However, if S encloses an uneven number of positive and negative monopoles, e.g. when Weyl nodes of opposite chirality lie at different energies, the quantity in Eq. (15) is non-zero and quantized. This result is an instance of the quantized non-linear photogalvanic effect [33], predicted in Ref. [34] for mirror-free Weyl semimetals; this alternative FGR-derivation highlights the deep connection of this quantized phenomenon to the quantization law in Eq. (1). In particular, it suggests how such an effect could be observed in a cold-atom realization of Weyl semimetals [63]: if the band velocities $v_{0,1}^a$ are known, measuring the local rate $\Delta\Gamma(\mathbf{k}; \omega)$ from wave-packets can lead to a quantized measurement through the integration in Eq. (14). Alternatively, one could probe the current [37] or the related center-of-mass velocity [22, 32, 65], giving directly access to the left hand side of Eq. (15).

Concluding remarks We anticipate that other drive protocols could lead to distinct quantized responses in higher spatial dimensions $D \geq 3$, offering the possibility of measuring higher-order Chern numbers through depletion rates [29, 66, 67]. Moreover, we emphasize that the general result in Eq. (1) could be generalized to interacting systems, as suggested by the real-space approach leading to Eq. (11); in this sense, the quantized DIR introduced in this work could be exploited to probe the topological order (e.g. many-body Chern number) of interacting systems [5]. Finally, we note that similar schemes could also probe chiral edge excitations [45], as well as the spin chirality of strongly-correlated states, as recently suggested in Ref. [68].

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